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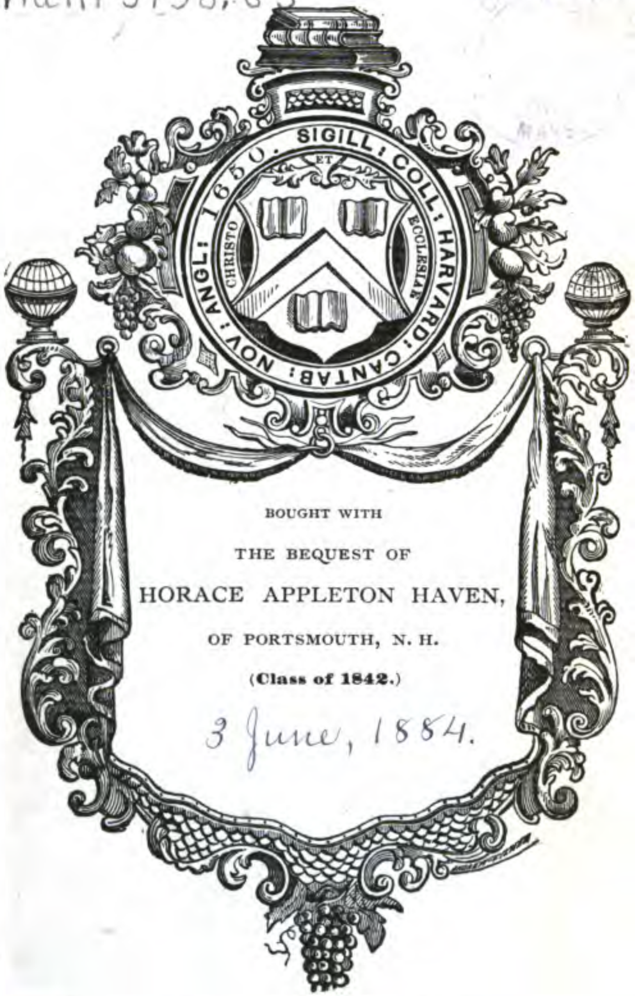
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3 June, 1884.

# A GEOMETRICAL TREATISE

ON

# CONIC SECTIONS.

Math
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A

# GEOMETRICAL TREATISE

ON

# CONIC SECTIONS.

WITH NUMEROUS EXAMPLES.

For the Use of Schools and Students in the Universities.

WITH
AN APPENDIX ON HARMONIC RATIO, POLES AND POLARS,
AND RECIPROCATION.

BY THE
REV. W. H. DREW, M.A.,
ST. JOHN'S COLLEGE, CAMBRIDGE,
PROFESSOR OF MATHEMATICS IN KING'S COLLEGE, LONDON.

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# PREFACE TO THE FIFTH EDITION.

In this Edition an Appendix has been added, in which an endeavour has been made to present the subject of Harmonic Ratio, Poles and Polars, and Reciprocation, in a form adapted to the wants of Students who approach these ideas for the first time.

A further collection of Problems has also been given, taken from Examination Papers of recent dates, and the work has thus been brought completely up to the requirements of the present time.

W. H. DREW.

King's College, London,
*June* 26, 1875.

# CONTENTS.

PAGE

INTRODUCTION . . . . . . . . . . . . . . . . . . . . . . . 1

## CHAPTER I.

THE PARABOLA . . . . . . . . . . . . . . . . . . . . . 3

PROBLEMS ON THE PARABOLA . . . . . . . . . . . . . 25

## CHAPTER II.

THE ELLIPSE . . . . . . . . . . . . . . . . . . . . . 29

PROBLEMS ON THE ELLIPSE . . . . . . . . . . . . . 63

## CHAPTER III.

THE HYPERBOLA . . . . . . . . . . . . . . . . . . . . 69

PROBLEMS ON THE HYPERBOLA . . . . . . . . . . . . . 107

## CHAPTER IV.

THE SECTIONS OF THE CONE . . . . . . . . . . . . . . 112

PROBLEMS ON THE SECTIONS OF THE CONE . . . . . . . . 123

ADDITIONAL PROBLEMS . . . . . . . . . . . . . . . . . 124

SECOND SERIES . . . . . . . . . . . . . . . . . . . . 139

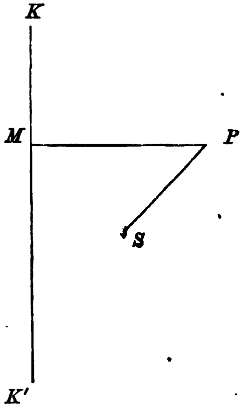
APPENDIX . . . . . . . . . . . . . . . . . . . . . . . 158

# CONIC SECTIONS.

## INTRODUCTION.

1. Def. The curve traced out by a point, which moves in such a manner that its distance from a given fixed point continually bears the same ratio to its distance from a given fixed line, is called a *Conic Section.*

The fixed point is called the *Focus*, and the fixed line the *Directrix.*



Thus if $S$ be the focus, and $KK'$ the directrix, and $P$ a point from which $PM$ is drawn at right angles to the directrix, the curve traced out by $P$ will be a *Conic Section,* provided $P$ move in such a manner that $SP$ always bears the same ratio to $PM$.

(1.) When the distance from the fixed point is equal to the distance from the fixed line, that is, when $SP$ is equal to $PM$, the *Conic Section* is called a *Parabola.*

(2.) When the distance from the fixed point is less than the distance from the fixed line, that is, when the ratio which

*SP* bears to *PM* is less than unity, the *Conic Section* is called an *Ellipse.*

(3.) When the distance from the fixed point is greater than the distance from the fixed line, that is, when the ratio which *SP* bears to *PM* is greater than unity, the *Conic Section* is called an *Hyperbola.*

2. The reason of the term *Conic Sections* being applied to these curves is that, when a *Cone* is intersected by a plane surface, the boundary of the section so formed will, *in general,* be one or other of these curves.

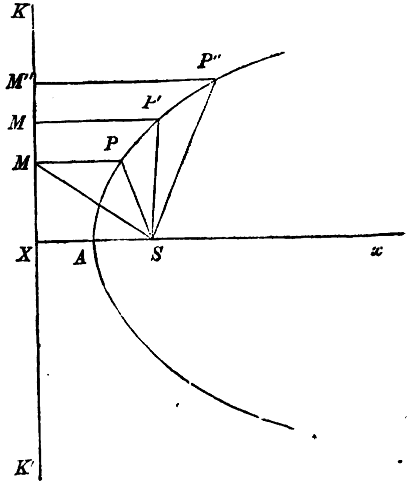
I propose to investigate the properties of the *Conic Sections* from the definitions given above, and afterwards to show in what manner a *Cone* must be divided by a plane in order that the curve of intersection may be a *Parabola, Ellipse,* or *Hyperbola.*

# CHAPTER I.

## THE PARABOLA.

### Prop. I.

3. The focus and directrix of a parabola being given, to find any number of points on the curve.



Let $S$ be the focus, and $KK'$ the directrix.

Draw $XSx$ at right angles to the directrix, and bisect the line $SX$ in $A$; then

$$\text{since } AS = AX,$$

$$\therefore A \text{ is a point on the curve.}$$

The point $A$ is called the *Vertex*, and the line $Ax$, with respect to which the curve is evidently symmetrical, is called the *Axis*.

On the directrix take any point $M$; join $SM$; and draw $MP$ at right angles to the directrix.

At the focus $S$ make the angle $MSP$ equal to the angle $SMP$; then

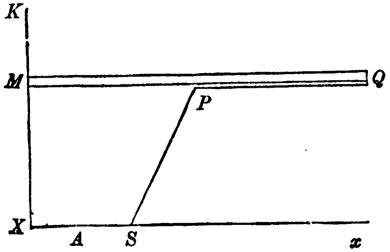
$$SP = PM,$$

$$\therefore P \text{ is a point on the curve.}$$

So by taking any number of points, $M'$, $M''$, on the directrix, we may obtain as many points, $P'$, $P''$, on the curve as we please, and the line which passes through $A$ and all these points will be the parabola whose focus is $S$ and directrix $KK'$.

COR. 1. As $M$ is taken further away from the point $X$, the line $SM$ and the angles $SMP$, $MSP$, and, consequently, the lines $SP$ and $PM$, continually increase. Hence, since $XM$ and $MP$ increase together, the curve recedes at the same time both from the axis and directrix; and since the angle $SMP$ can never exceed a right angle, and the lines $SP$ and $MP$ will therefore always meet, it is evident that there is no limit to the distance to which the curve may extend on both sides of the axis.

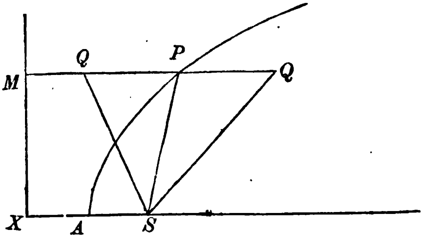
COR. 2. The parabola may be described practically in the following manner.



Let $S$ be the focus and $KX$ be the directrix; and let a rigid bar $QM$, having a string of the same length as itself fastened at one end $Q$, be made to slide parallel to the axis with the other end $M$ on the directrix; then if the other end of the string be fastened at the focus, and the string be kept stretched by means of the point of a pencil at $P$, in contact with the bar, since $SP$ will always be equal to $PM$, it is evident that the point $P$ will trace out the parabola.

### PROP. II.

4. The distance of any point inside the parabola from the focus is less than its distance from the directrix; and the distance of any point outside the parabola from the focus is greater than its distance from the directrix.



(1.) Let $Q$ be a point inside the parabola.

Draw $QM$ at right angles to the directrix, meeting the parabola in $P$; join $SP$; then

$$\text{since } SP = PM,$$
$$\therefore SP \text{ and } PQ = QM.$$
$$\text{But } SP \text{ and } PQ > SQ,$$
$$\therefore QM > SQ.$$

(2.) Let $Q$ be a point outside the parabola.

Draw $MQ$ at right angles to the directrix, and produce it to meet the parabola in $P$; join $SP$; then

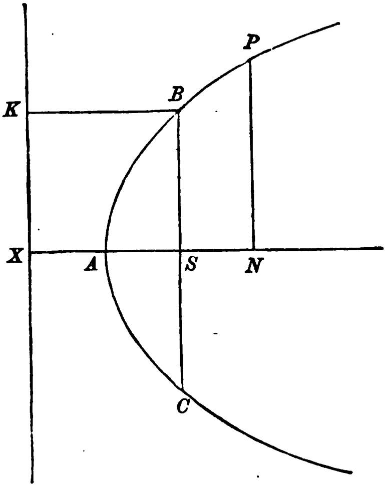
$$\text{since } SQ \text{ and } QP > SP,$$
$$\text{and } SP = PM,$$
$$\therefore SQ \text{ and } QP > PM,$$
$$\therefore SQ > QM.$$

COR. Conversely a point will be inside or outside the parabola according as its distance from the focus is less or greater than its distance from the directrix.

5. DEF. The line $PN$ (*see fig. Prop.* III.) drawn at right angles to the axis from the point $P$ in the curve is called the *Ordinate* of the point $P$, and the line $AN$ the *Abscissa.* The double ordinate $BC$ drawn through the focus, and terminated both ways by the curve, is called the *Latus Rectum.*

## Prop. III.

The Latus Rectum $BC = 4AS$.



Draw $BK$ at right angles to the directrix.

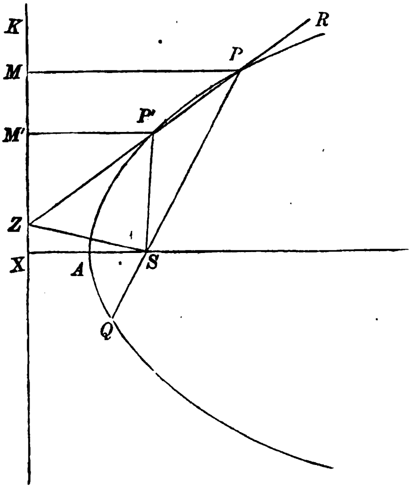
$$\text{Then } SB = BK = SX = 2AS,$$
$$\therefore BC = 4AS.$$

6. Def. If a point $P'$ be taken on the parabola (*see fig. Prop.* IV.) near to $P$, and $PP'$ be joined, the line $PP'$ produced, in the limiting position which it assumes when $P'$ is made to approach indefinitely near to $P$, is called the *Tangent* to the parabola at the point $P$.

## Prop. IV.

If the tangent to the parabola at any point $P$ intersect the directrix in the point $Z$; then $SZ$ will be at right angles to $SP$.

Let $P'$ be a point on the parabola near to $P$.



Draw the chord $PP'$ and produce it to meet the directrix in $Z$; join $SZ$.

Draw $PM$, $P'M'$ at right angles to the directrix; join $SP$, $SP'$; and produce $PS$ to meet the parabola in $Q$.

Then, since the triangles $ZMP$, $ZM'P'$ are similar,

$$\therefore ZP : ZP' :: MP : M'P',$$
$$:: SP : SP',$$

$\therefore SZ$ bisects the angle $P'SQ$. (*Euclid*, VI. *Prop.* A.) Whitworth Bk II Prop 3.

Now when $P'$ is indefinitely near to $P$, and $PP'$ becomes the tangent at the point $P$, the angle $PSP'$ becomes indefinitely small, while the angle $QSP'$ approaches two right angles, and therefore the angle $P'SZ$, which is half of the angle $P'SQ$, becomes ultimately a right angle.

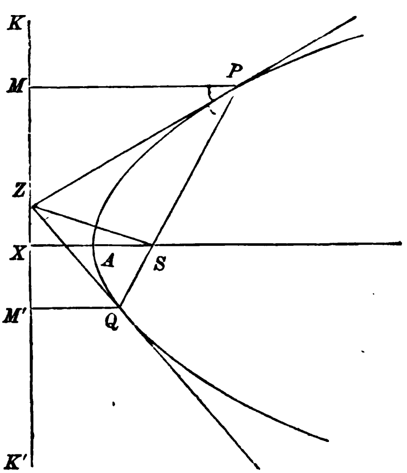
Hence, when $PZ$ is the tangent,

the angle $ZSP$ is a right angle,
or $SZ$ is perpendicular to $SP$.

Cor. Conversely, if $SZ$ be drawn at right angles to $SP$, meeting the directrix in $Z$, and $PZ$ be joined, $PZ$ will be a tangent at $P$.

### Prop. V.

7. The tangent at any point $P$ of a parabola bisects the angle between the focal distance $SP$, and the perpendicular $PM$ on the directrix.



Let the tangent at $P$ meet the directrix in the point $Z$; join $SZ$; then since the angle $ZSP$ is a right angle, (*Prop.* IV.)

$$\therefore ZS^2 + SP^2 = PZ^2.$$

$$\text{Also } ZM^2 + MP^2 = PZ^2.$$

$$\therefore ZS^2 + SP^2 = ZM^2 + MP^2.$$

$$\text{But } SP = PM,$$

$$\therefore ZS = ZM.$$

Now in the triangles $ZPS$, $ZPM$,

$\therefore ZP, PS = ZP, PM$, each to each,

and $ZS = ZM$,

$\therefore$ the angle $SPZ =$ the angle $MPZ$;

or $PZ$ bisects the angle $SPM$.

COR. 1. If $ZP$ be produced to $R$, then the angle $SPR$ = the angle $MPR$.

COR. 2. It is evident that the tangent at the vertex $A$ is perpendicular to the axis.

## PROP. VI.

8. The tangents at the extremities of a focal chord intersect at right angles in the directrix.

Let $PSQ$ be a focal chord, and let the tangent at $P$ meet the directrix in $Z$.

Join $SZ$; then

the angle $ZSP$ is a right angle, (*Prop.* IV.)

and ∴ also the angle $ZSQ$ is a right angle,

∴ $ZQ$ is the tangent at $Q$, (*Prop.* IV. *Cor.*)

or the tangents at the extremities of the focal chord $PSQ$ intersect in the directrix.

Again, draw $PM$, $QM'$ at right angles to the directrix; then

since $MP$, $PZ$ = $SP$, $PZ$, each to each,

and the angle $MPZ$ = the angle $SPZ$,

∴ the angle $MZP$ = the angle $SZP$,

∴ the angle $SZP$ is half of the angle $SZM$.

So the angle $SZQ$ is half of the angle $SZM'$,

∴ the angle $PZQ$ is half of the two $SZM$ and $SZM'$.

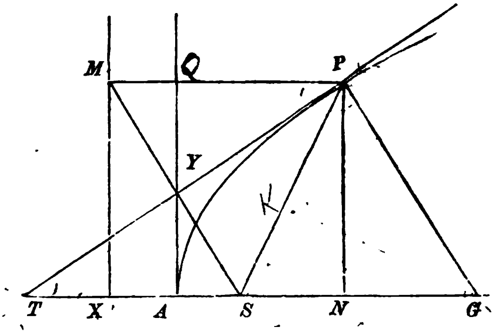
But the angles $SZM$ and $SZM'$ = two right angles,

∴ the angle $PZQ$ is a right angle,

or the tangents at the extremities of a focal chord intersect *at right angles* in the directrix.

## Prop. VII.

**9. If the tangent at any point $P$ of a parabola meet the axis produced in the point $T$, and $PN$ be the ordinate of the point $P$, then $NT = 2AN$.**



Join $SP$, and draw $PM$ at right angles to the directrix; then

$$\therefore \text{ the angle } SPT = \text{the angle } MPT = \text{the angle } STP,$$

$$\therefore ST = SP.$$

$$\text{But } SP = PM = XN,$$

$$\therefore ST = XN.$$

$$\text{But } AS = AX,$$

$$\therefore \text{ the remainder } AT = \text{the remainder } AN,$$

$$\therefore NT = 2AN.$$

Def. The line $NT$ is called the *Subtangent*.

10. Def. The line $PG$, drawn at right angles to $PT$, is called the *Normal* at the point $P$, and $NG$ the *Subnormal*.

## Prop. VIII.

If the normal at the point $P$ of a parabola meet the axis in the point $G$, then $NG = 2AS$.

Since the angle $SPG$ = the complement of the angle $SPT$,
and the angle $SGP$ = the complement of the angle $STP$,
and also the angle $SPT$ = the angle $STP$, (*Prop.* VII.)
$\therefore$ the angle $SPG$ = the angle $SGP$,
$\therefore SG = SP$.
But $SP = PM = XN$,
$\therefore SG = XN$.

Taking away the common part $SN$,
the remainder $NG = SX = 2AS$.

### PROP. IX.

11. If $PN$ be an ordinate to the parabola at the point $P$; then $PN^2 = 4AS \,.\, AN$.

Since $TPG$ is a right angle, and $PN$ perpendicular to $TG$;
$\therefore$ $PN$ is a mean proportional between $TN$ and $NG$;
or $PN^2 = TN \,.\, NG$. (*Euclid*, VI. 8 *Cor.*)
But $TN = 2AN$, (*Prop.* VII.)
and $NG = 2AS$, (*Prop.* VIII.)
$\therefore PN^2 = 4AS \,.\, AN$.

### PROP X.

12. If the tangent at any point $P$ intersect the tangent at the vertex in $Y$, then $SY$ will bisect $PT$ at right angles, and will be a mean proportional between $SA$ and $SP$.

Draw $PN$ at right angles to the axis; then
since $AY$ is parallel to $PN$,
$\therefore TY : YP :: TA : AN$.
But $AT = AN$, (*Prop.* VII.)
$\therefore TY = PY$;
and $\therefore SY$, $YP = SY$, $YT$, each to each,
and $SP = ST$, (*Prop.* VII.)
$\therefore$ the angle $SYP$ = the angle $SYT$,
$\therefore SY$ is perpendicular to $PT$.

Again, since $TYS$ is a right angle, and $YA$ perpendicular to $ST$,

$\therefore$ $SY$ is a mean proportional between $ST$ and $SA$;

or $SY^2 = ST \cdot SA$. (*Euclid*, VI. 8 *Cor.*)

But $ST = SP$, (*Prop.* VII.)

$\therefore$ $SY^2 = SP \cdot SA$.

Cor. If $PM$ be drawn at right angles to the directrix, and $MY$ be joined, then

since $SP$, $PY = MP$, $PY$, each to each,

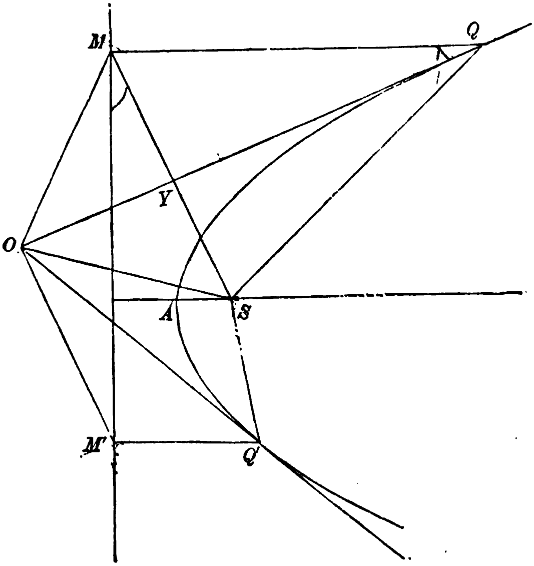
and the angle $SPY =$ the angle $MPY$, (*Prop.* V.)

$\therefore$ the angle $SYP =$ the angle $MYP$,

$\therefore$ $SY$ and $YM$ are in the same straight line.

### Prop. XI.

13. To draw a pair of tangents to a parabola from an external point.



Let $O$ be the given external point.

Join $OS$, and with centre $O$ and radius $OS$ describe a circle, cutting the directrix in $M$ and $M'$, which it will always do, on whichever side of the directrix $O$ is situated, since $O$ is nearer to the directrix than to the focus. (*Prop.* II.)

Draw $MQ$ and $M'Q'$ parallel to the axis meeting the parabola in $Q$ and $Q'$.

Join $OQ$, $OQ'$; these will be the tangents required.

Join $SQ$ and $SQ'$; then

$\therefore$ $OQ$, $QS$ = $OQ$, $QM$, each to each,

and $OS = OM$,

$\therefore$ the angle $OQS$ = the angle $OQM$,

$\therefore$ $OQ$ is the tangent at $Q$. (*Prop.* V.)

So $OQ'$ is the tangent at $Q'$.

## PROP. XII.

14. If from a point $O$ a pair of tangents $OQ$ and $OQ'$ be drawn to a parabola, the triangles $OSQ$, $OSQ'$ will be similar, and $OS$ will be a mean proportional between $SQ$ and $SQ'$.

Join $SM$, cutting $OQ$ at right angles (*Prop.* X. *Cor.*) in the point $Y$; then

since the angle $SQO$ = the angle $MQO$, (*Prop.* V.)

and the angle $MQO$ = the angle $SMM'$,

each of these angles being the complement of the angle $QMY$,

$\therefore$ the angle $SQO$ = the angle $SMM'$.

But the angle $SMM'$ at the circumference is half the angle $SOM'$ at the centre, and is therefore equal to the angle $SOQ'$.

$\therefore$ the angle $SQO$ = the angle $SOQ'$.

So the angle $SOQ$ = the angle $SQ'O$,

$\therefore$ the remaining angle $OSQ$ = the remaining angle $OSQ'$.

And therefore the triangle $OSQ$ is similar to the triangle $OSQ'$

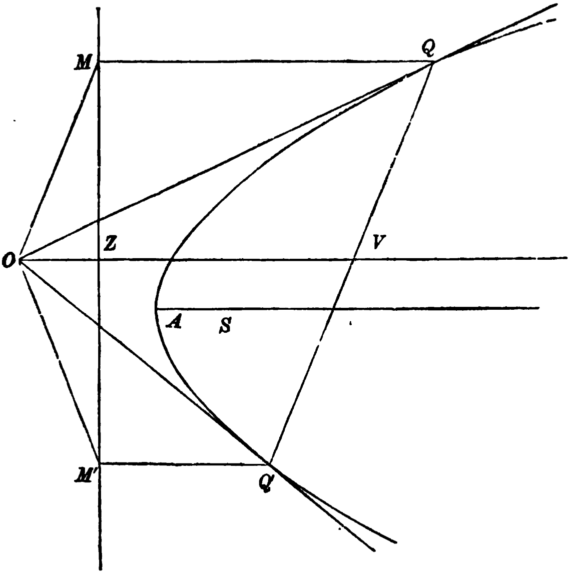
$$\therefore SQ : SO :: SO : SQ',$$

$$\therefore SQ . SQ' = SO^2.$$

or $SO$ is a mean proportional between $SQ$ and $SQ'$.

### Prop. XIII.

15. If a pair of tangents $OQ$, $OQ'$ be drawn to a parabola, and $OV$ be drawn parallel to the axis meeting $QQ'$ in $V$, then $QQ'$ shall be bisected in $V$.



Draw $QM$, $Q'M'$ at right angles to the directrix.

Join $OM$, $OM'$; and let $OV$ meet $MM'$ in $Z$.

Then, since $OM = OM'$, (*Prop.* XI.)

$\therefore$ the angle $OMZ =$ the angle $OM'Z$,

and the angle $OZM =$ the angle $OZM'$,

and the side $OZ$ is common to the triangles $OZM$, $OZM'$,

$$\therefore\ MZ = M'Z.$$

And because the lines $QM$, $ZV$, $Q'M'$ are parallel,

$$\therefore\ QV : Q'V :: MZ : M'Z.$$

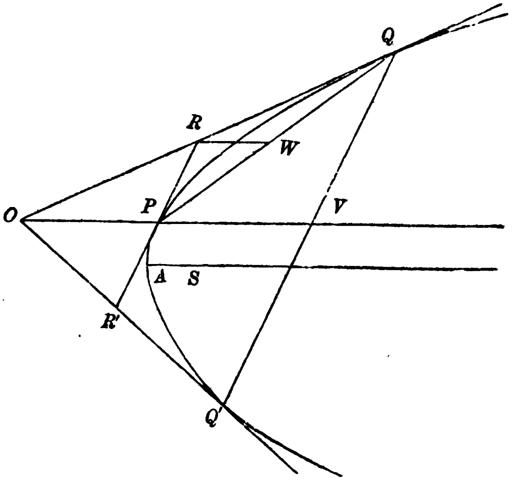
$$\text{But } MZ = M'Z,$$

$$\therefore\ QV = Q'V,$$

$$\therefore\ QQ' \text{ is bisected in } V.$$

### Prop. XIV.

16. If from a point $O$ a pair of tangents $OQ$, $OQ'$, be drawn to a parabola, and $OV$ be drawn parallel to the axis meeting the parabola in $P$, and $QQ'$ in $V$, then the tangent at $P$ will be parallel to $QQ'$ and $OV$ will be bisected in $P$.



Draw the tangent $RPR'$ meeting $OQ$, $OQ'$ in $R$ and $R'$.

Join $PQ$, and draw $RW$ parallel to the axis, meeting $PQ$ in $W$;

Then, by the last Proposition,

$$PW = WQ.$$

And because $RW$ is parallel to $OP$,

$$\therefore\ OR : RQ :: PW : WQ.$$

$$\text{But } PW = WQ,$$

$$\therefore\ OR = RQ;$$

$$\text{so } OR' = R'Q',$$

$$\therefore\ OR : RQ :: OR' : R'Q',$$

$$\therefore\ RR' \text{ is parallel to } QQ'.$$

Again, since $PR$ is parallel to $QV$,

$$\therefore\ OP : PV :: OR : RQ.$$

$$\text{But } OR = RQ,$$

$$\therefore\ OP = PV.$$

Cor. From this it is manifest that if any number of parallel chords be drawn in a parabola, their middle points will all lie on the line parallel to the axis which passes through the point where the tangent drawn parallel to the chord meets the parabola.

Def. Any line $PV$, drawn from a point $P$ in the parabola parallel to the axis, is called a *Diameter.*

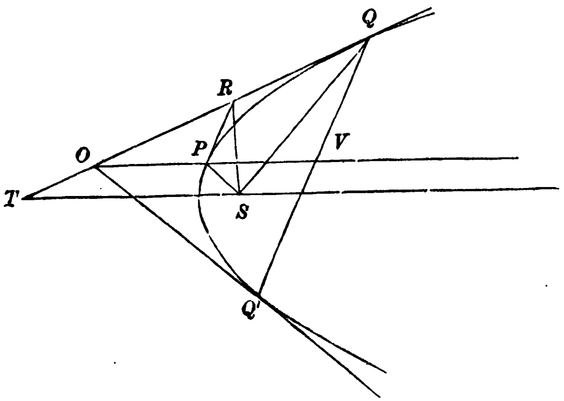
The point $P$ is called the *Vertex* of the diameter $PV$; and the tangent at $P$ the *Tangent at the Vertex.*

The diameter consequently bisects all chords parallel to the tangent at the vertex, and the tangents at the extremities of any chord will intersect in the diameter corresponding to that chord.

Def. A line $QV$, drawn parallel to the tangent at $P$ from a point $Q$ in the curve, is called the *Ordinate* to the diameter $PV$.

## Prop. XV.

17. If $QV$ be an ordinate to the diameter $PV$, then $QV^2 = 4 . SP . PV$.



Produce $QV$ to meet the parabola in $Q'$; and draw the tangents $QO$, $Q'O$, meeting $VP$ produced in the point $O$. (*Prop.* XIV.)

Also let the tangent at $P$ meet $OQ$ in $R$, and join $SP$, $SR$, and $SQ$. Now since from the point $R$ two tangents $RP$, $RQ$ are drawn to the parabola, the triangle $RPS$ is similar to the triangle $RSQ$, (*Prop.* XII.)

$\therefore$ the angle $SRP =$ the angle $SQR$.

But the angle $SQR =$ the angle $STQ$, (*Prop.* VII.)

$=$ the angle $POR$,

$\therefore$ the angle $SRP =$ the angle $POR$,

and the angle $SPR =$ the angle $OPR$, (*Prop.* V. *Cor.* 1.)

$\therefore$ the remaining angle $RSP =$ the remaining angle $ORP$,

$\therefore$ the triangle $SPR$ is similar to the triangle $POR$,

$$\therefore SP : PR :: PR : PO,$$

$$\therefore PR^2 = SP \cdot PO,$$

$$= SP \cdot PV. \quad (\textit{Prop. } \text{XIV.})$$

Again, since $QV$ is parallel to $PR$,

$$\therefore QV : PR :: OV : OP.$$

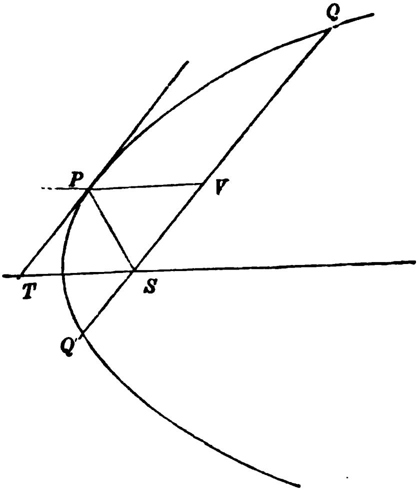
But $OV = 2\,OP$, (*Prop.* XIV.)

$$\therefore QV = 2PR,$$
$$\therefore QV^2 = 4PR^2,$$
$$= 4SP \cdot PV.$$

18. **Def.** The double ordinate to the diameter $PV$, drawn parallel to the tangent at $P$, and passing through the focus, is called the *Parameter* of *the diameter* $PV$.

### Prop. XVI.

The parameter of the diameter $PV = 4 \cdot SP$.

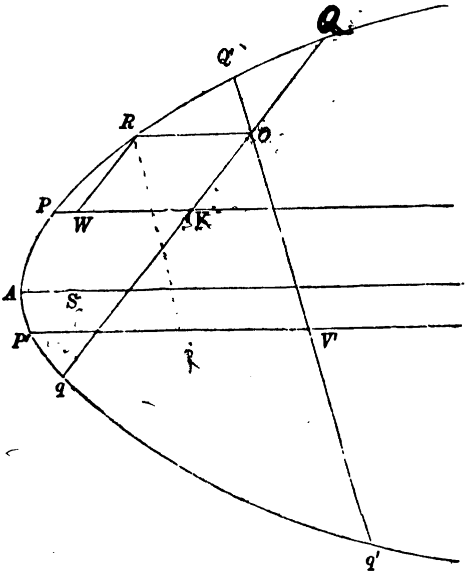


Draw $QSQ'$ through the focus parallel to the tangent at $P$, and let the tangent at $P$ meet the axis produced in $T$; then

$$QV^2 = 4\,SP \cdot PV. \quad (\textit{Prop.}\ \text{XV.})$$
$$\text{But } PV = ST = SP, \quad (\textit{Prop.}\ \text{VII.})$$
$$\therefore QV^2 = 4SP^2;$$
$$\text{or } QV = 2SP,$$
$$\therefore QQ' = 4SP.$$

## Prop. XVII.

19. If two chords of a parabola intersect one another, the rectangles contained by their segments are in the ratio of the parameters of the diameters which bisect the chords.



Let the chords $Qq$, $Q'q'$ intersect one another in the point $O$.

Bisect $Qq$, $Q'q'$ in $V$ and $V'$; and draw the diameters $PV$, $P'V'$ parallel to the axis.

Also, through $O$ draw $OR$ parallel to $PV$; and through $R$ draw $RW$ parallel to $QV$.

Now, since $Qq$ is divided equally in $V$ and unequally in $O$,

$$\therefore QO \, . \, Oq = QV^2 - OV^2, \quad (\textit{Euclid}, \text{ II. } 5)$$
$$= QV^2 - RW^2,$$
$$= 4SP \, . \, PV - 4SP \, . \, PW, \quad (\textit{Prop.} \text{ XV.})$$
$$= 4SP \, . \, RO.$$

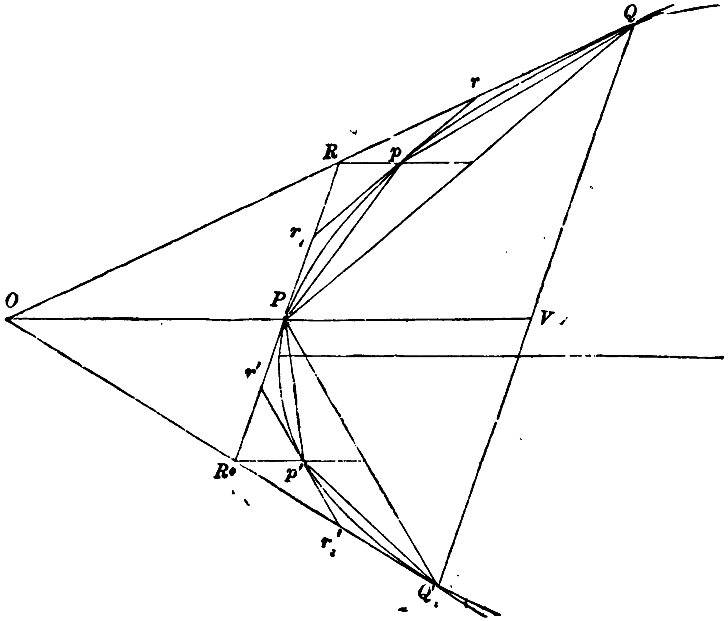
So $Q'O \, . \, Oq' = 4SP' \, . \, RO$.

Hence $QO \, . \, Oq : Q'O \, . \, Oq' :: 4SP : 4SP'$.

By *Euclid*, II. 6, the same may be proved to be true if the point $O$ be without the parabola.

## Prop. XVIII.

20. If from an external point $O$ a pair of tangents $OQ$, $OQ'$ be drawn to the parabola, and the chord $QQ'$ be joined, the area of the figure bounded by $QQ'$ and the curve is two-thirds of the triangle $QOQ'$.



Draw the diameter $OV$ meeting the curve in $P$; and let the tangent at $P$ meet $OQ$, $OQ'$ in $R$ and $R'$.

Join $QP$, $Q'P$; then

$$\text{since } OR = RQ,$$

$$\therefore \text{ the triangle } OPR = \tfrac{1}{2} \text{ the triangle } OPQ,$$
$$= \tfrac{1}{2} \text{ the triangle } VPQ.$$

$$\text{So the triangle } OPR' = \tfrac{1}{2} \text{ the triangle } VPQ',$$

$$\therefore \text{ the triangle } ORR' = \tfrac{1}{2} \text{ the triangle } PQQ'.$$

Again, if through $R$ and $R'$ we draw the diameters $Rp$, $R'p'$; and at the points $p$ and $p'$ draw the tangents $rpr_{\prime}$, $r'p'r'_{\prime}$, we can prove in the same manner as before that

$$\text{the triangle } Rrr_{\prime} = \tfrac{1}{2} \text{ the triangle } QpP,$$

$$\text{and the triangle } R'r'r'_{\prime} = \tfrac{1}{2} \text{ the triangle } Q'p'P.$$

Continuing in this manner to form new triangles by drawing diameters at the points $r$, $r_{\prime}$, and $r'r'_{\prime}$, and tangents at the points where these diameters meet the curve, we can prove that the exterior triangles formed by the tangents are the halves of the interior triangles formed by joining the points of contact with the extremities of the chords.

And the same will hold however the number of the triangles be increased.

Hence the sum of all the exterior triangles will be equal to half the sum of all the interior triangles.

Now when the number of the triangles is increased indefinitely, the sum of the exterior triangles will represent the exterior figure $OQPQ'$, and the sum of the interior triangles the area of the interior figure $QPQ$. Hence

$$\text{the area of the figure } OQPQ' = \tfrac{1}{2} \text{ the area of the figure } QPQ'$$

$$\therefore \text{ area of the figure } OQPQ' = \tfrac{1}{3} \text{ the area of triangle } QOQ',$$

$$\therefore \text{ area of the figure } QPQ' = \tfrac{2}{3} \text{ the area of triangle } QOQ'.$$

21. Def. If with a point $O$ on the normal at $P$ as centre and $OP$ as radius, a circle be described touching the parabola at $P$ and cutting it in $Q$; then when the point $Q$ is made to approach indefinitely near to $P$, the circle is called the *Circle of Curvature* at the point $P$. (*See fig. Prop.* XIX.)

## PROP. XX.

**The chord of the circle of curvature at a point $P$ of a parabola, drawn parallel to the axis $= 4SP$.**

Let $PT$ be the tangent, and $PG$ the normal at the point $P$.

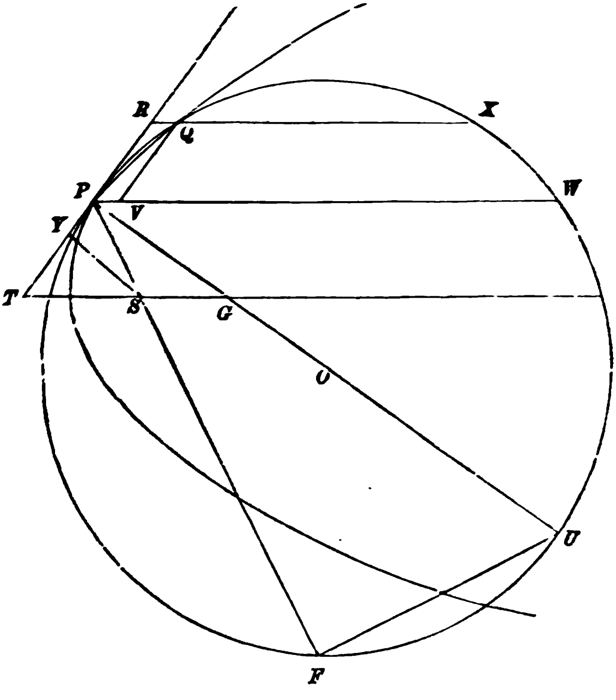
With centre $O$ and radius $OP$ describe a circle cutting the parabola in the point $Q$.

Draw $RQX$ parallel to the axis meeting the circle in $X$ and the tangent at $P$ in $R$.

Also draw $QV$ parallel to $PR$, and $PW$ parallel to the axis; then

since $RP$ touches the circle at $P$,

$\therefore RQ \, . \, RX = PR^2$. (*Euclid*, III. 36.)



But $PR^2 = QV^2 = 4SP \,.\, PV$, (*Prop.* XV.)

$$\therefore RQ \,.\, RX = 4SP \,.\, PV.$$

$$\text{But } RQ = PV.$$

$$\therefore RX = 4SP.$$

Now when the circle becomes the circle of curvature at $P$, the points $R$ and $Q$ move up to and coincide with $P$, and the lines $RX$ and $PW$ become equal.

Hence the chord of the circle of curvature parallel to the axis $= 4SP$.

Cor. 1. If $PU$ be the diameter of the circle of curvature, and $PF$ the chord through the focus; then

since the angle $FPU =$ the angle $WPU$, (*Prop.* VIII.)

$$\therefore PF = PW = 4SP.$$

Cor. 2. If $SY$ be drawn at right angles to $PT$; then

the triangle $PFU$ is similar to $SYP$,

$$\therefore PU : PF :: SP : SY,$$

$$\text{or } PU : 4SP :: SP : SY.$$

### Prop. XX.

If $QVQ'$ be any ordinate to the diameter $PV$, the circle described through the three points $P$, $Q$, $Q'$ will intersect the parabola in a fourth point, which depends only upon the position of $P$.

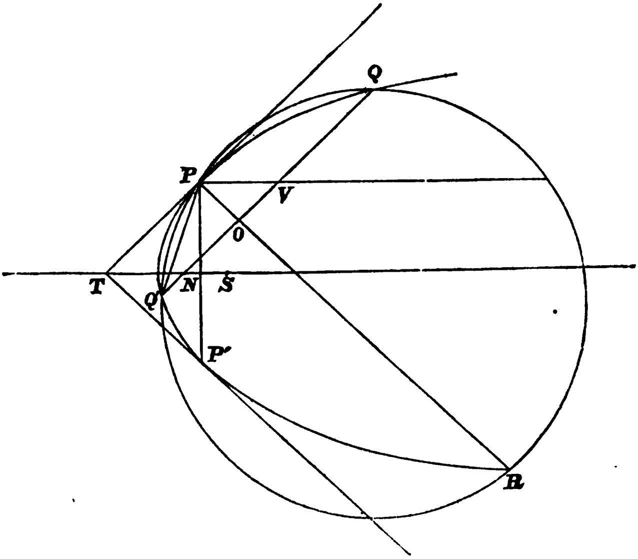
Draw the ordinate $PN$, and produce it to meet the parabola in $P'$; then,

since the subtangent $= 2 \,.\, AN$. (*Prop.* VII.)

the tangents at $P$ and $P'$ will meet the axis in the same point $T$.

Draw $PR$ parallel to $TP'$, meeting the parabola in $R$, and $QQ'$ in $O$; then

$PO \,.\, OR : QO \,.\, OQ' :: SP' : SP$. (*Prop.* XVII.)



But $SP = ST = SP'$, (*Prop.* VII.)

$$\therefore PO \,.\, OR = QO \,.\, OQ'.$$

Hence by the converse of *Euclid* III. *Prop.* 35, the point $R$ is on the circle which passes through $P$, $Q$, $Q'$.

COR. 1. Since $TP$ and $TP'$ are equally inclined to the axis, the lines $QQ'$, $PR$, which are parallel respectively to $TP$ and $TP'$, are also equally inclined to the axis.

COR. 2. When the point $V$ is brought indefinitely near to $P$, $QQ'$ coincides with the tangent to the parabola at $P$, and becomes also a tangent to the circle at $P$, since $Q$ and $Q'$ are indefinitely near to each other. The circle therefore becomes the circle of curvature at the point $P$.

Hence if $PR$ be drawn parallel to the tangent at $P'$, or be equally inclined to the axis with $PT$, it will meet the parabola in the point where the circle of curvature at $P$ intersects the parabola.

## PROBLEMS ON THE PARABOLA.

1. The diameter of the circle described about the triangle $BAC$ is equal to $5AS$. (*See fig. Prop.* III.)

2. If from the point $G$, $GK$ be drawn at right angles to $SP$, then $PK = 2AS$. (*See fig. Prop.* VII.)

3. If the triangle $SPG$ is equilateral, then $SP$ is equal to the latus rectum. (*See fig. Prop.* VII.)

4. $PQ$ is a common tangent to a parabola and the circle described on the latus rectum as diameter; prove that $SP$ and $SQ$ make equal angles with the latus rectum.

5. Prove that $PY \cdot PZ = SP^2$, and that $PY \cdot YZ = AS \cdot SP$. (*See fig. Prop.* VII.)

6. If $PL$ be drawn at right angles to $AP$, meeting the axis in $L$, and $PN$ be the ordinate of $P$, then $NL = 4AS$.

7. The tangent at any point $P$ of a parabola meets the directrix and latus rectum produced in points equally distant from the focus.

8. Prove that $NY = TY$, and that $TP \cdot TY = TS \cdot TN$. (*See fig. Prop.* VII.)

9. If a circle be described about the triangle $SPN$, the tangent to it from $A = \frac{1}{2} PN$. (*See fig. Prop.* VII.)

10. If the ordinate of a point $P$ bisect the subnormal of $P'$, the ordinate of $P$ is equal to the normal of $P'$.

11. If from any point on the tangent to a parabola a line be drawn touching the parabola, the angle between this line and the line to the focus from the same point is constant.

12. A circle and parabola have the same vertex and axis. $BA'C$ is the double ordinate of the parabola which touches the circle at $A'$, the extremity of the diameter through the vertex $A$. $PP'$ is any other ordinate of the parabola parallel to this, meeting the axis in $N$, and $AB$ produced in $R$; prove that the rectangle $RP . RP'$ is proportional to the square of the tangent drawn from $N$ to the circle.

13. Draw a parabola to touch a given circle at a given point, and such that its axis may touch the same circle in another given point.

14. If from the point of contact of a tangent to a parabola a chord be drawn, and another line be drawn parallel to the axis meeting the chord, tangent, and curve, this line will be divided by them in the same ratio as it divides the chord.

15. If the diameter $PV$ meet the directrix in $O$, and the chord drawn through the focus parallel to the tangent at $P$ in $V$, prove that $VP = PO$.

16. Prove that the locus of the intersection of a diameter $PV$ with the chord drawn through the focus parallel to the tangent at $P$ is a parabola.

17. If a circle and parabola have a common tangent at $P$, and intersect in $Q$ and $R$; and $QV$, $UR$ be drawn parallel to the axis of the parabola meeting the circle in $V$ and $U$ respectively, then $VU$ is parallel to the tangent at $P$.

18. $AB$ and $AC$ are two lines at right angles to each other. From a fixed point $C$ on $AC$, $CR$ is drawn parallel to $AB$. On $AR$, produced if necessary, $P$ is taken such that the perpendicular $PN$ upon $AB$ is equal to $CR$. Prove that the curve traced out by $P$ is a parabola.

19. If from a point $P$ of a circle $PC$ be drawn to the centre; and $R$ be the middle point of the chord $PQ$ drawn parallel to a fixed diameter $ACB$, then the curve traced out by the intersection of $CP$ and $AR$ is a parabola.

20. If two equal tangents $OQ$, $OQ'$, be cut by a third tangent, their alternate segments are equal.

21. $E$ is the centre of the circle described about the triangle $OQQ'$. Prove that the circle described about the triangle $QEQ'$ will pass through the focus. (*See fig. Prop.* XIII.)

22. $PSp$ is any focal chord of a parabola. Prove that $AP$, $Ap$ will meet the latus rectum in two points $Q$, $q$, whose distances from the focus are equal to the ordinates of $p$ and $P$.

23. $PSp$ is a focal chord of a parabola, $RDr$ the directrix meeting the axis in $D$; and $Q$ any point on the curve. Prove that if $QP$, $Qp$ be produced to meet the directrix in $R$, $r$, half the latus rectum is a mean proportional between $DR$, $Dr$.

24. $OP$ and $OQ$ are two tangents to a parabola. On $QO$ produced, $OQ'$ is taken equal to $OQ$; prove that $OS \, . \, PQ' = OP \, . \, OQ$.

25. If $QD$ be drawn at right angles to the diameter $PV$, then $QD^2 = 4AS \, . \, PV$.

26. If through any point $O$ on the axis of a parabola a chord $POQ$ be drawn, and $PM$, $QN$ be the ordinates of the points $P$ and $Q$, prove that $AM \, . \, AN = AO^2$.

27. If $AP$ and $AQ$ be drawn at right angles to each other from the vertex of a parabola, and $PM$, $QN$ be the ordinates of $P$ and $Q$, prove that the latus rectum is a mean proportional between $AM$ and $AN$.

28. $OAP$ is the sector of a circle whose centre is $O$. If the radius $OA$ remain fixed while the angle $AOP$ changes the centre of the circle inscribed in the sector, $AOP$ will trace out a parabola.

29. $QSQ'$ is a focal chord parallel to $AP$; $PN$, $QM$, $Q'M'$ are the ordinates of $P$, $Q$, and $Q'$. Prove that $SM^2 = AM \, . \, AN$ and that $MM' = AP$.

30. $PQ$, $PQ'$ are drawn from any point $P$ cutting the ordinates $Q'V'$, $QV$ in $R'$ and $R$, prove that $VR$ is to $V'R'$ in the triplicate ratio of $QV$ to $Q'V'$.

31. On a chord of a parabola as diameter a circle is described cutting the parabola again in two points. If these points be joined, the portion of the axis between the two chords is equal to the latus rectum.

32. If $OQ$, $OQ'$ be a pair of tangents to a parabola, and the chord $QQ'$ be a normal to the curve at $Q$, then $OQ$ is bisected by the directrix.

33. Two equal parabolas having the same focus and their axes in contrary directions intersect at right angles.

34. The radius of curvature at the extremity of the latus rectum is equal to twice the normal.

35. If from any point $P$ of a parabola $PF$ and $PH$ be drawn making equal angles with the normal $PG$, then $SG^2 = SF \,.\, SH$.

36. If a triangle be inscribed in a parabola, the points when the sides produced meet the tangents at the opposite angles are in the same straight line.

37. If the tangents $OQ$, $OQ'$ be cut by a third tangent in $R$, $R'$, prove that

$$OR : RQ :: R'Q' : OR'.$$

38. If from the vertex of a parabola chords be drawn at right angles to one another, and on them a rectangle be described, the curve traced out by the further angle is a parabola.

39. Prove that $2PY$ is a mean proportional between $AP$ and the chord of the circle of curvature at the point $P$ of the parabola drawn through the vertex $A$. (*See fig. Prop.* VII.)

40. If a circle described upon the chord of a parabola as diameter meet the directrix it also touches it, and all chords for which this is possible intersect in a point.

41. If a parabola *roll* upon another equal parabola, the vertices originally coinciding, the focus traces out the directrix.

42. The circle of curvature at the extremity of the latus rectum intersects the parabola on the diameter of curvature passing through the point of contact.

# CHAPTER II.

## THE ELLIPSE.

22. Def. The *Ellipse* is the curve traced out by a point which moves in such a manner that its distance from a given fixed point continually bears the same ratio, *less than unity*, to its distance from a given fixed line. (*See Introduction.*)

### Prop. I.

The focus and directrix of an ellipse being given, to find any number of points on the curve.

Let $S$ be the focus, and $MX$ the directrix.

Draw $SX$ at right angles to the directrix, and divide $SX$ in the point $A$, so that $SA$ may be to $AX$ in the given fixed ratio less than unity; then

$A$ is a point on the curve.

On $XS$ produced take a point $A'$, such that

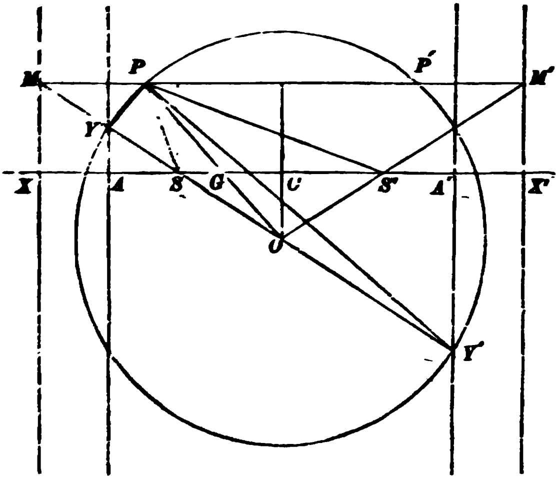
$$SA' : A'X :: SA : AX;$$

then $A'$ will also be a point on the curve.

On the directrix take *any* point $M$; and through $M$ and $S$ draw the line $MYSY'$ meeting $AY$ and $A'Y'$, drawn at right angles to $AA'$ in the points $Y$ and $Y'$.

On $YY'$ as diameter describe a circle, and draw $MPP'$ parallel to $AA'$, cutting the circle in the points $P$ and $P'$;

$P$ and $P'$ will be points on the ellipse.



Join $PY$, $PY'$, $SP$; then since

$SY : YM :: SA : AX$, (*Euclid*, VI. 2.)

and $SY' : Y'M :: SA' : A'X$, (*Euclid*, VI. 2.)

$\therefore SY : YM :: SY' : Y'M$;

or, alternately,

$SY : SY' :: YM : Y'M$,

and the angle $YPY'$ in a semicircle is a right angle,

$\therefore PY$ bisects the angle $SPM$,*

$\therefore SP : PM :: SY : YM$,

$:: SA : AX$.

So we may show that

$SP' : P'M :: SY : YM$,

$:: SA : AX$,

$\therefore P$ and $P'$ are points on the curve.

---

* For, if not, make the angle $YPs$ equal to $YPM$; then

$sY : YM :: sP : PM$. (*Euclid*, VI. 3.)

And since, if $PY$ bisect $sPM$, $PY'$, being at right angles to $PY$, also bisects the angle $sPM'$,

$\therefore sY' : Y'M :: sP : PM$. (*Euclid*, VI. A.)

Hence $sY : YM :: sY' : Y'M$,

or $sY : sY' :: YM : Y'M$,

$\therefore$ the points $S$ and $s$ coincide.

In the same way, by taking other points on the directrix, we may obtain as many more points on the curve as we please.

Cor. 1. Since, corresponding to every point $P$ on the curve, there is a point $P'$ situated in precisely the same manner with respect to $A'Y'$ as $P$ is with respect to $AY$, it is clear that if we make $A'S'$ equal to $AS$, and $A'X'$ equal to $AX$, and draw $X'M'$ at right angles to $AX'$, the curve could be equally well described with $S'$ as focus and $M'X'$ as directrix.

The ellipse is therefore symmetrical, not only with respect to the line $AA'$, but also with respect to the line $OC$ drawn through the middle point of $YY'$ at right angles to and bisecting $AA'$.

Cor. 2. The line $OP$ will bisect the angle $SPS'$.

Let $OP$ meet $SS'$ in $G$. Produce $MP$ to meet $X'M'$ in $M'$, and draw $OM'$ passing through the focus $S'$; then

$$SP : PM :: S'P : PM',$$

or, alternately, $SP : S'P :: PM : PM'$. (1)

Again, $SG : PM :: S'G : PM'$,

or, alternately, $SG : S'G :: PM : PM'$, (2)

$\therefore$ from (1) and (2)

$$SP : S'P :: SG : S'G,$$

$\therefore$ $PG$ bisects the angle $SPS'$. (*Euclid*, VI. 3.)

It will be shown hereafter (*Prop.* XI.) that the normal to the ellipse at the point $P$ also bisects the angle $SPS'$. Hence the ellipse and circle have the same tangent at the point $P$.

The ellipse will consequently touch all the infinite series of circles which can be described in the same manner as the one in the figure by taking different points on the directrix.

### PROP. II.

23. If $C$ be the middle point of $AA'$, then $CA$ is a mean proportional between $CS$ and $CX$,

or $CS \,.\, CX = CA^2$. (*See fig. Prop.* III.)

Since $SA' : A'X :: SA : AX$.

Alternately $SA' : SA :: A'X : AX$,

$\therefore\ SA' + SA : SA :: A'X + AX : AX$;

or $AA' : SA :: XX' : AX$,

$\therefore\ AA' : XX' :: SA : AX$,

or $CA : CX :: SA : AX$. (1)[*]

Again, $SA' : SA :: A'X : AX$;

$\therefore\ SA' - SA : SA :: A'X - AX : AX$;

or $SS' : SA :: AA' : AX$.

Alternately $SS' : AA' :: SA : AX$;

or $CS : CA :: SA : AX$. (2)

Hence from (1) and (2)

$$CA : CX :: CS : CA,$$

$$\therefore\ CA^2 = CX \,.\, CS;$$

or $CA$ is a mean proportional between $CS$ and $CX$.

COR. Since the three lines $CS$, $CA$, $CX$ are proportional, therefore, by the definition of duplicate ratio and *Euclid*, VI. 20 *Cor.*

$$CS : CX : CS^2 : CA^2. \quad (3)$$

### PROP. III.

24. If $P$ be any point on the ellipse, then

$$SP + S'P = AA'.$$

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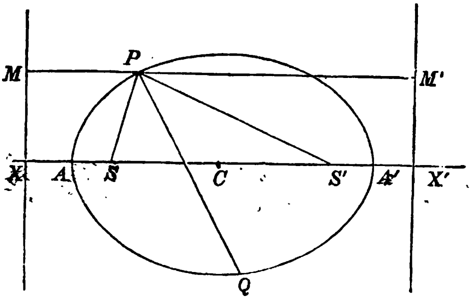
[*] N.B. The results (1), (2), (3) should be remembered, as they will frequently be referred to.



$$\begin{array}{rl}
\text{Since } SP : PM :: & SA : AX, \\
\text{and } SA : AX :: & AA' : XX', \quad (\textit{Prop.}\ \text{II.}) \\
\therefore SP : PM :: & AA' : XX', \\
\text{So } S'P : PM' :: & AA' : XX', \\
\therefore SP + S'P : PM + PM' :: & AA' : XX'. \\
\text{But } PM + PM' = MM' = & XX', \\
\therefore SP + S'P = & AA'.
\end{array}$$

COR. 1. By means of this property the ellipse may be practically described and the form of the curve determined.

Let a string, equal in length to $AA'$, have its ends fastened to two points $S$ and $S'$; and let it be kept stretched by means of the point of a pencil at $P$; then since $SP + S'P$ will be always equal to $AA'$, the point $P$ will trace out the ellipse.

COR. 2. The line $AA'$ is the longest line that can be drawn in the ellipse.

For, if any other line $PQ$ be drawn, then

$$\begin{array}{r}
SP + SQ > PQ, \\
\text{and } S'P + S'Q > PQ, \\
\therefore SP + S'P + SQ + S'Q > 2PQ, \\
\text{or } AA' > PQ.
\end{array}$$

25. DEF. If $BCB'$ be drawn at right angles to $ACA'$, meeting the ellipse in $B$ and $B'$, it will be seen further on (*Prop.* XIII. *Cor.* 2) that $BCB'$ is the shortest chord that can be drawn through the *centre* of the ellipse. (*See fig. Prop.* IV.)

$AA'$ is called the *Major Axis* and $BB'$ the *Minor Axis* of the ellipse.

In most geometrical treatises the ellipse is defined as the curve traced out by a point which moves in such a manner that the sum of its distances from two fixed points is always the same; but it appears that the properties of the curve are more clearly exhibited by defining it in a manner analogous to the parabola, and deducing *immediately* from that definition the property in question.

Having now shown that one definition necessarily includes the other, we are at liberty in our future investigations to make use of whichever property is most convenient.

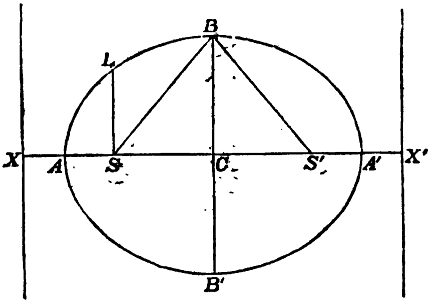
### PROP. IV.

26. If $BC$ be the semi-minor axis of the ellipse, then

$$BC^2 = CA^2 - CS^2;$$

and if $SL$ be the semi-latus rectum,

$$SL \cdot AC = BC^2.$$



Join $SB$, $S'B$; then

$$\text{since } SB + S'B = AA', \quad (\textit{Prop.}\ \text{III.})$$
$$\text{and that } SB = S'B,$$
$$\therefore SB = AC.$$
$$\text{But } BC^2 = SB^2 - CS^2,$$
$$\therefore BC^2 = CA^2 - CS^2,$$

Again, $SL : SX :: SA : AX,$

$$:: CS : CA, \quad (\textit{Prop.}\ \text{II.})$$

$$\therefore SL \,.\, AC = CS \,.\, SX,$$

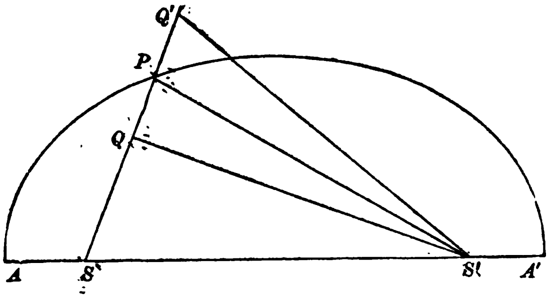
$$= CS \,.\, CX - CS^2, \quad (\textit{Euclid},\ \text{II. 3,})$$

$$= CA^2 - CS^2, \quad (\textit{Prop.}\ \text{II.})$$

$$= BC^2.$$

## PROP. V.

27. The sum of the distances of *any* point from the foci of an ellipse will be less or greater than $AA'$ according as the point is inside or outside the ellipse.



(1) Let $Q$ be a point inside the ellipse.

Join $SQ$, $S'Q$; and produce $SQ$ to meet the ellipse in $P$; join $S'P$; then

$$\text{since } S'P + QP > S'Q,$$

$$\therefore S'P + SP > S'Q + SQ.$$

$$\text{But } S'P + SP = AA', \quad (\textit{Prop.}\ \text{III.})$$

$$\therefore SQ + SQ' < AA'.$$

(2) Let $Q'$ be a point outside the ellipse.

Join $SQ$, $S'Q$, and let $SQ$ meet the ellipse in the point $P$; join $S'P$; then

$$\text{since } S'Q + QP > S'P,$$

$$\therefore S'Q + SQ > SP + S'P,$$

$$\text{But } SP + S'P = AA', \quad (\textit{Prop.}\ \text{III.})$$

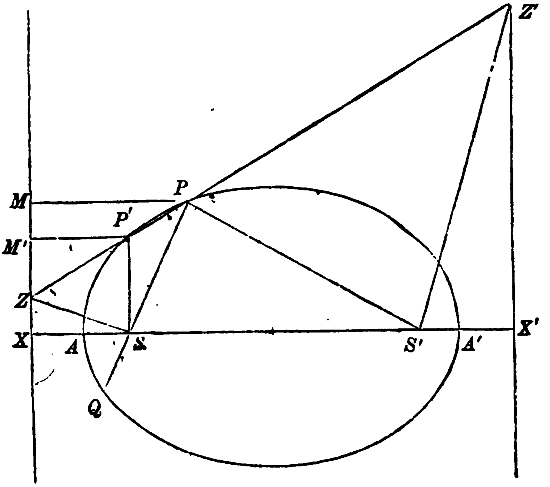
$$\therefore SQ + S'Q > AA'.$$

COR. Conversely, a point will be inside or outside the ellipse, according as the sum of its distances from the foci is less or greater than $AA'$.

28. DEF. If a point $P'$ be taken on the ellipse near to $P$, (*see fig. Prop.* VI.) and $PP'$ be joined, the line $PP'$ produced, in the limiting position which it assumes when $P'$ is made to approach indefinitely near to $P$, is called the *Tangent* to the ellipse at the point $P$.

## PROP. VI.

If the tangent to the ellipse at any point $P$ intersect the directrix in the point $Z$, and if $S$ be the focus corresponding to the directrix on which $Z$ is situated, then $SZ$ will be at right angles to $SP$.



Let $P'$ be a point on the ellipse near to $P$.

Draw the chord $PP'$, and produce it to meet the directrix in $Z$; join $SZ$.

Draw $PM$, $P'M'$ at right angles at the directrix, and join $SP$, $SP'$.

Produce $PS$ to meet the ellipse in the point $Q$; then since the triangles $ZMP$, $ZM'P'$ are similar,

$$\therefore\ ZP : ZP' :: MP : M'P',$$
$$:: SP : SP',$$

$\therefore$ $SZ$ bisects the angle $P'SQ$. (*Euclid*, VI. *Prop.* A.)

Now when $P'$ is indefinitely near to $P$, and $PP'$ becomes the tangent at the point $P$, the angle $PSP'$ becomes indefinitely small, while the angle $QSP'$ approaches two right angles; and therefore the angle $ZSP'$ being half of the angle $P'SQ$, becomes ultimately a right angle.

Hence when $PZ$ becomes the tangent at the point $P$,

the angle $ZSP$ is a right angle,
or $SZ$ is perpendicular to $SP$.

COR. 1. Conversely, if $SZ$ be drawn at right angles to $SP$, meeting the directrix in $Z$, and $PZ$ be joined, $PZ$ will be the tangent at $P$.

COR. 2. If $ZP$ be produced to meet the other directrix on the point $Z'$ and $S'Z'$ be joined, then

$S'Z'$ is at right angles to $S'P$.

COR. 3. The tangents at the extremities of the latus rectum or double ordinate through the focus meet the axis produced in the point $X$.

### PROP. VII.

The tangent to the ellipse at any point $P$ makes equal angles with the focal distances $SP$ and $S'P$.

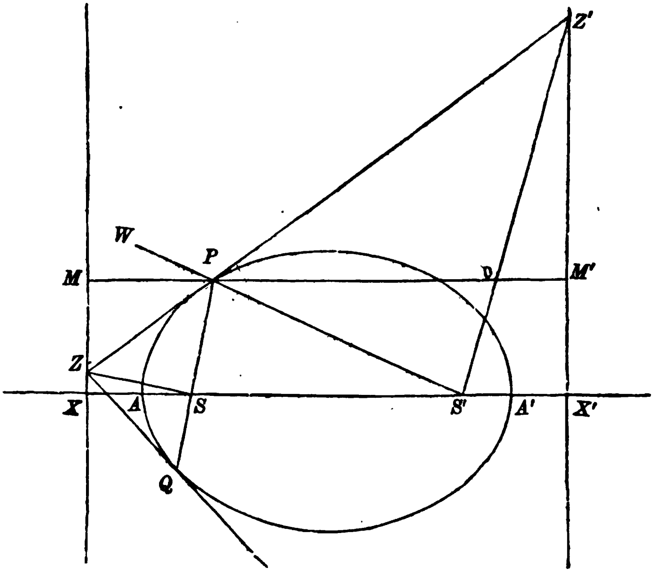
Let the tangent at $P$ meet the directrices in $Z$ and $Z'$.

Draw $MPM'$ at right angles to the directrices, meeting them in $M$ and $M'$ respectively; join $SZ$, $S'Z'$; then

$$SP : PM :: S'P : PM';$$

and since the triangles $MPZ$, $M'PZ'$, are similar,

$$PM : PZ :: PM' : PZ',$$
$$\therefore\ SP : PZ :: S'P : PZ'. \quad (\textit{Ex æquali.})$$



Now in the triangles $SPZ$, $S'PZ'$, because the sides about the angles $SPZ$, $S'PZ'$ are proportional, and the angles $PSZ$, $PS'Z'$ are equal, being right angles, and the angles $SZP$, $S'Z'P$ are each less than a right angle,

∴ the triangles $SPZ$ and $S'PZ'$ are similar, (*Euclid*, VI. 7)

∴ the angle $SPZ$ = the angle $S'PZ'$.

Cor. If $S'P$ be produced to $W$; then

the angle $SPZ$ = the angle $WPZ$.

## Prop. VIII.

omit

The tangents at the extremities of a focal chord intersect in the directrix.

Let $PSQ$ be a focal chord, and let the tangent at $P$ meet the directrix in $Z$.

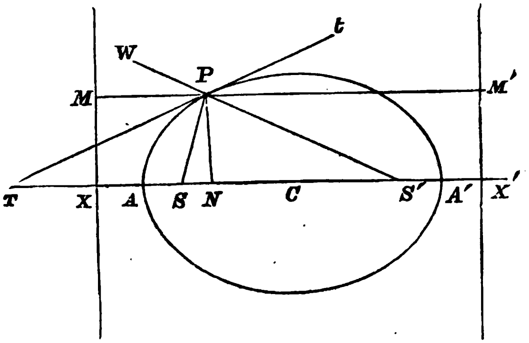
Join $SZ$; then

the angle $ZSP$ is a right angle, (*Prop.* VI.)
and $\therefore$ also the angle $ZSQ$ is a right angle,
$\therefore$ $ZQ$ is the tangent at $Q$; (*Prop.* VI. *Cor.* 1)
or the tangents at the extremities of a focal chord intersect in the directrix.

### Prop. IX.

29. If the tangent at $P$ meet the axis major produced in $T$, and $PN$ be the ordinate of the point $P$, then

$$CT \, . \, CN = CA^2.$$

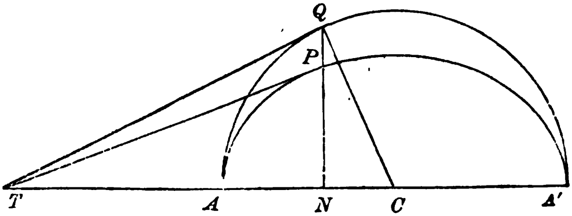


Draw $MPM'$ parallel to the axis major meeting the directrices in $M$ and $M'$; and produce $S'P$ to $W$; then, since $PT$ bisects the angle $SPW$, (*Prop.* VII. *Cor.*)

$$\therefore S'T : ST :: S'P : SP, \quad (\textit{Euclid}, \text{VI. A.})$$
$$:: PM' : PM,$$
$$:: X'N : XN,$$
$$\therefore S'T + ST : S'T - ST :: X'N + XN : X'N - XN;$$
$$\text{or } 2CT : 2CS :: 2CX : 2CN,$$
$$\text{or } CT : CS :: CX : CN,$$
$$\therefore CT \, . \, CN = CS \, . \, CX,$$
$$= CA^2. \quad (\textit{Prop.} \text{ II.})$$

## PROP. X.

If on the major axis of an ellipse as diameter a circle be described and a common ordinate $NPQ$ be drawn meeting the ellipse in $P$ and the circle in $Q$, then the tangents to the ellipse and circle respectively at the points $P$ and $Q$ will meet the major axis produced in the same point.



Let the tangent to the ellipse at $P$ meet the major axis produced in $T$; join $CQ$, $QT$; then, by the last Proposition,

$$CT \cdot CN = CA^2 = CQ^2,$$

$\therefore$ the angle $CQT$ is a right angle.

And therefore $QT$ is the tangent to the circle at $Q$; or the tangents at $P$ and $Q$ meet the major axis produced in the same point $T$.

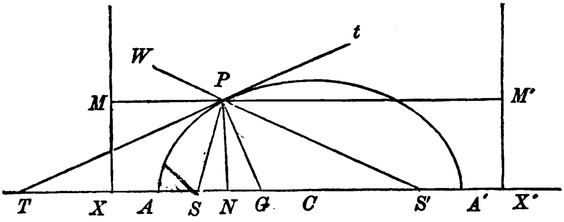
The circle described on $AA'$ as diameter is called the *Auxiliary Circle* on account of the important aid that it affords in investigating the properties of the ellipse.

30. DEF. The line $PG$, drawn at right angles to the tangent $PT$, is called the *Normal* to the ellipse at the point $P$.

## PROP. XI.

If the normal at $P$ meet the major axis in the point $G$; then

$$SG : SP :: CS : CA,$$



Since $PG$ is at right angles to $TPt$,
$\therefore$ the angle $GPT =$ the angle $GPt$.
But the angle $SPT =$ the angle $S'Pt$, (*Prop.* VII.)
$\therefore$ the angle $SPG =$ the angle $S'PG$,
or $PG$ bisects the angle $SPS'$,

$$\therefore SG : S'G :: SP : S'P, \quad (\textit{Euclid}, \text{VI. } 3.)$$
$$\therefore SG : SG + S'G :: SP : SP + S'P;$$
$$\text{or } SG : SS' :: SP : AA';$$
$$\text{or } SG : SP :: SS' : AA';$$
$$\text{or } SG : SP :: CS : CA.$$

Cor. Hence also,

$$S'G : S'P :: CS : CA.$$

### Prop. XII.

31. If the normal at $P$ meet the major axis in $G$, and $PN$ be the ordinate at the point $P$, then (*see fig. Prop.* XI.)

$$NG : NC :: BC^2 : AC^2.$$

Draw $MPM'$ parallel to the axis meeting the directrices in $M$ and $M'$; join $SP$, $S'P$; then, since $PG$ bisects the angle $SPS'$, (*Prop.* XI.)

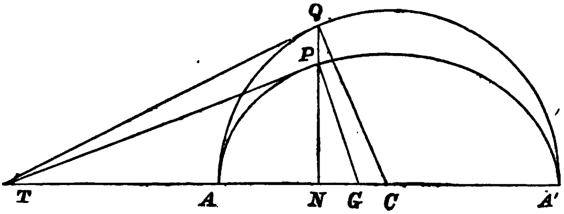
$$\therefore S'G : SG :: S'P : SP,$$
$$:: PM' : PM,$$
$$:: X'N : XN,$$
$$\therefore S'G - SG : S'G + SG :: X'N - XN : X'N + XN;$$
$$\text{or } 2CG : SS' :: 2CN : XX'.$$

$$\begin{array}{rcl} \text{Alternately, } 2CG : 2CN & :: & SS' : XX'; \\ \text{or } CG : CN & :: & CS : CX, \\ & :: & CS^2 : CA^2, \quad (\textit{Prop.}\ \text{II. } \textit{Cor.}) \\ \therefore CN - CG : CN & :: & CA^2 - CS^2 : CA^2; \\ \text{or } NG : CN & :: & BC^2 : AC^2. \end{array}$$

### PROP. XIII.

32. If $PN$ be the ordinate of any point $P$ on the ellipse; then

$$PN^2 : AN \,.\, A'N :: BC^2 : AC^2.$$



Produce $NP$ to meet the auxiliary circle in the point $Q$, and draw the tangents $PT$, $QT$ meeting the major axis produced in the point $T$. (*Prop.* X.)

Join $CQ$, and let the normal at $P$ meet the ellipse in $G$ then, by the last Proposition,

$$NG : CN :: BC^2 : AC^2.$$

And rectangles of the same altitude are to another as their bases,

$$\therefore TN \,.\, NG : TN \,.\, CN :: BC^2 : AC^2;$$

$$\text{or } PN^2 : QN^2 :: BC^2 : AC^2. \quad (\textit{Euclid},\ \text{VI. 8, } \textit{Cor.})$$

$$\text{But } QN^2 = AN \,.\, A'N,$$

since the angle $AQA'$ in a semicircle is a right angle,

$$\therefore PN^2 : AN \,.\, A'N :: BC^2 : AC^2.$$

COR. 1. Also $PN : QN :: BC : AC$.

This result is the basis of many of the future Propositions of the ellipse.

COR. 2. Since $PN^2 : QN^2 :: BC^2 : AC^2$,

$$\therefore PN^2 : AC^2 - CN^2 :: BC^2 : AC^2,$$

$$\therefore PN^2 : AC^2 - CN^2 - PN^2 :: BC^2 : AC^2 - BC^2,$$

$$\text{or } PN^2 : AC^2 - CP^2 :: BC^2 : AC^2 - BC^2.$$

Now $PN^2$ is always less than $BC^2$,

$\therefore$ $CP^2$ is greater than $BC^2$,

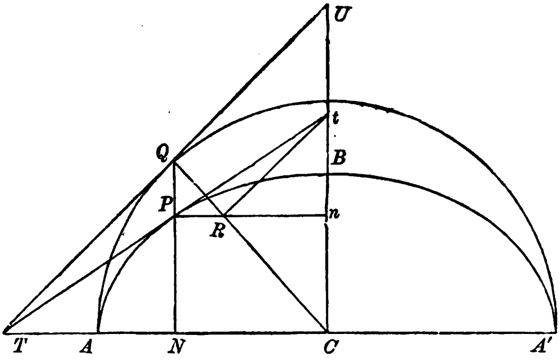
$\therefore$ $BC$ is the shortest line that can be drawn to the ellipse from the centre.

Omit!

## PROP. XIV.

If the tangent at any point $P$ of an ellipse meet the minor axis $CB$ produced in $t$, and $Pn$ be drawn at right angles to $CB$; then

$$Ct \cdot Cn = BC^2,$$



Draw the common ordinate $NPQ$ to the ellipse and the auxiliary circle; and let the tangents at $P$ and $Q$ to the ellipse and circle respectively meet the major axis produced in $T$ (*Prop.* X.) and the minor axis produced in $t$ and $U$.

Join $CQ$ meeting $Pn$ in $R$; then since $PR$ is parallel to $CN$,

$$CR : CQ :: PN : QN.$$

$$:: BC : AC. \quad (\textit{Prop.}\ \text{XIII. } \textit{Cor.}\ 1.)$$

$$\text{But } CQ = AC,$$
$$\therefore CR = BC. \quad (1)$$

Again, joining $Rt$,

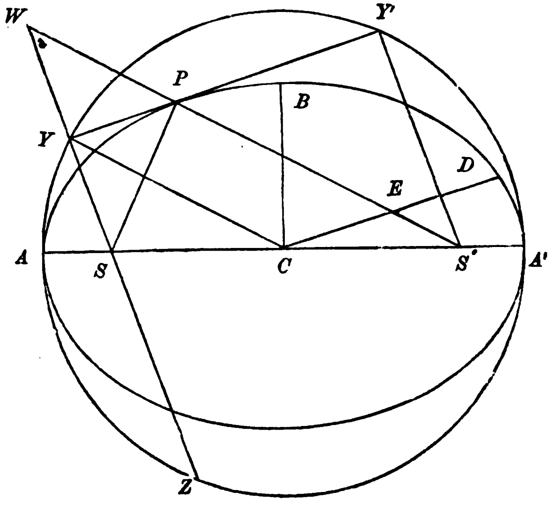
$$Ct : CU :: PN : QN,$$
$$:: CR : CQ,$$
$$\therefore Rt \text{ is parallel to } QU,$$
$$\therefore CRt \text{ is a right angle,}$$
$$\therefore Ct \,.\, Cn = CR^2 \quad (\textit{Euclid}, \text{VI. 8, } \textit{Cor.})$$
$$\text{But } CR = BC, \quad (1)$$
$$\therefore Ct \,.\, Cn = BC^2.$$

This proposition also admits of a demonstration similar to that given for the corresponding property of the hyperbola.

### PROP. XV.

33. If from the foci $S$ and $S'$, $SY$ and $S'Y'$ are drawn at right angles to the tangent at $P$, then $Y$ and $Y'$ are on the circumference of the auxiliary circle, and

$$SY \,.\, S'Y' = BC^2.$$



Join $SP$, $S'P$, and produce $SY$ and $S'P$ to meet in $W$; and join $CY$; then

since the angle $SPY$ = the angle $WPY$, (*Prop.* VII. *Cor.*)

and the angle $SYP$ = the angle $WYP$,

and the side $PY$ is common to the triangles $SPY$, $WPY$,

$\therefore$ the triangle $SPY$ = the triangle $WPY$ in all respects,

$$\therefore SP = PW,$$
$$\therefore SP + S'P = S'W.$$
$$\text{But } SP + S'P = AA', \quad (\textit{Prop.}\ \text{III.})$$
$$\therefore S'W = AA'.$$

Again $\because SC = S'C$ and $SY = YW$,

$$\therefore SC : S'C :: SY : YW,$$
$$\therefore CY \text{ is parallel to } S'W,$$
$$\therefore CY : S'W :: CS : SS',$$
$$\therefore CY = \tfrac{1}{2} S'W = CA.$$
$$\text{So } CY' = CA,$$

$\therefore$ $Y$ and $Y'$ are points on the auxiliary circle.

Next let $YS$ be produced to meet the auxiliary circle in $Z$, and join $ZY'$; then

since the angle $ZYY'$ is a right angle,

$\therefore$ $ZY'$ passes through the centre $C$,

$\therefore$ the angle $SCZ$ = the angle $S'CY'$.

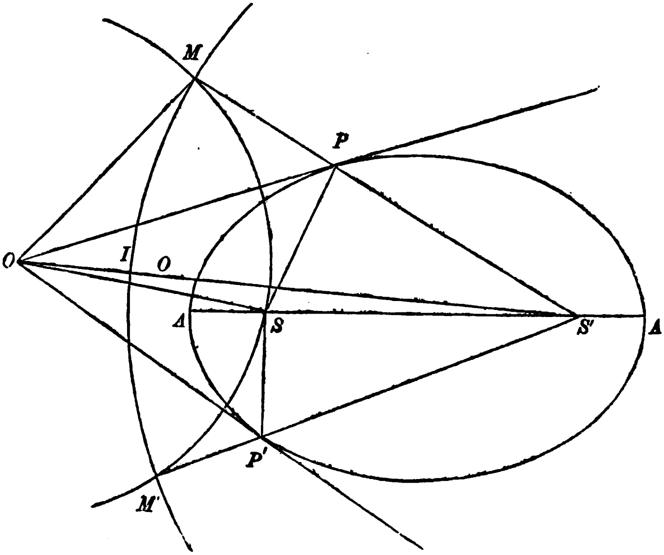
$$\therefore SZ = S'Y',$$
$$\begin{aligned}\therefore SY \,.\, S'Y' &= SY \,.\, SZ,\\ &= AS \,.\, A'S, \quad (\textit{Euclid},\ \text{III. } 35)\\ &= CA^2 - CS^2, \quad (\textit{Euclid},\ \text{II. } 5)\\ &= BC^2. \quad (\textit{Prop.}\ \text{IV.})\end{aligned}$$

COR. If $CD$ be drawn parallel to the tangent at $P$, meeting $S'P$ in $E$; then since the figure $CYPE$ is a parallelogram,

$$\therefore PE = CY = AC.$$

### Prop. XVI.

34. To draw a pair of tangents to an ellipse from an external point $O$.



With centre $S'$ and radius equal to $AA'$ describe a circle.

Join $OS$, $OS'$; and let $SO$ or $S'O$ produced meet the circle in the point $I$.

Now, if $O$ be a point outside the circle $MIM'$, it is evident that $OS$ is greater than $OI$; and if $O$ be inside the circle,

$$\text{since } OS + OS' > AA' \text{ or } S'I, \quad (\textit{Prop.}\ \text{V.})$$
$$\therefore OS > OI.$$

With centre $O$ and radius $OS$ describe another circle cutting the former in the points $M$ and $M'$, which it will always do, since $OS$ is greater than $OI$.

Join $S'M$, $S'M'$, meeting the ellipse in the points $P$ and $P'$.

Join $OP$, $OP'$; these will be the tangents required.

Join $SP$, $SP'$; then, since

$$SP + S'P = AA' = S'M,$$

$$\therefore\ SP = PM.$$

And $\because\ SP,\ PO = MP,\ PO$, each to each,

and $OS = OM$,

$\therefore$ the angle $OPS$ = the angle $OPM$,

$\therefore$ $OP$ is the tangent at $P$. (*Prop.* VII. *Cor.*)

So $OP'$ is the tangent at $P'$.

end.

## PROP. XVII.

If from a point $O$ a pair of tangents $OP$, $OP'$ be drawn to an ellipse, then $OP$ and $OP'$ will subtend equal angles at either focus.

Join $SP$, $S'P$; $SP'$, $S'P'$; and produce $S'P$, $S'P'$ to $M$ and $M'$, making $PM$ equal to $SP$, and $P'M'$ equal to $SP'$.

Join $OM$, $OM'$; $OS$, $OS'$.

Then since $OP,\ PS = OP,\ PM$, each to each.

and the angle $OPS$ = the angle $OPM$, (*Prop.* VII. *Cor.*)

$$\therefore\ OS = OM,$$

and the angle $OSP$ = the angle $OMP$.

So $OS = OM'$,

and the angle $OSP'$ = the angle $OM'P'$,

$$\therefore\ OM = OM'.$$

Again, $\because\ S'M = S'P + SP = AA'$,

and $S'M' = S'P' + SP' = AA'$,

$$\therefore\ S'M = S'M',$$

And $\because\ OS',\ S'M = OS',\ S'M'$, each to each,

and $OM = OM'$,

$\therefore$ the angle $OS'M$ = the angle $OS'M'$,

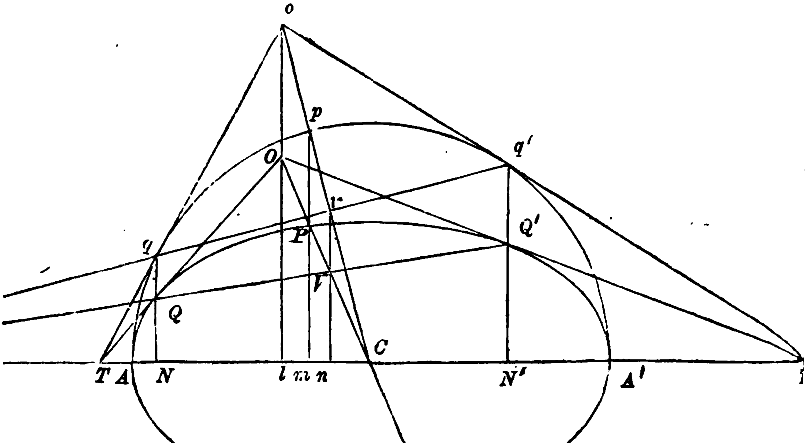
and the angle $OMS'$ = the angle $OM'S'$.

But the angle $OMS'$ = the angle $OSP$,
and the angle $OM'S'$ = the angle $OSP'$,
∴ the angle $OSP$ = the angle $OSP'$,
∴ $OP$ and $OP'$ subtend equal angles at either focus.

## Prop. XVIII.

35. If from an external point $O$ a pair of tangents $OQ$, $OQ'$ be drawn to an ellipse, and $CO$ be joined meeting the chord $QQ'$ in $V$, and the ellipse in $P$; then

(1.) $QQ'$ will be bisected in $V$.
(2.) The tangent at $P$ will be parallel to $QQ'$.
(3.) $CP$ will be a mean proportional between $CV$ and $CO$.



Produce $OQ$, $OQ'$ to meet the major axis produced in $T$ and $T'$.

Draw the ordinates $NQ$, $N'Q'$, and produce them to meet the circle in $q$ and $q'$.

Then $Tq$ and $T'q'$ will be tangents to the auxiliary circle. (*Prop.* X.)

Let $Tq$ and $T'q'$ be produced to meet in $o$; join $Co$ meeting the chord $qq'$ in $v$, and the circle in $p$.

Now, since the corresponding ordinates of the ellipse and auxiliary circle are in the constant ratio of $BC$ to $AC$, the three lines $ol$, $pm$, $vn$ drawn at right angles to $AA'$ will pass through the points $O$, $P$, $V$ respectively.

For, according as $O$ is the point where $ol$ meets $TQ$ or $T'Q'$ we shall have

$$\begin{aligned} lO : lo &:: NQ : Nq, \\ &:: BC : AC; \\ \text{or } lO : lo &:: N'Q' : N'q', \\ &:: BC : AC, \end{aligned}$$

$\therefore$ $Oo$ is perpendicular to $AA'$.

So $Pp$ and $Vv$ are perpendicular to $AA'$,

$\therefore$ $Oo$, $Pp$, $Vv$ are parallel.

Hence (1.) $QV : VQ' :: qv : vq'$.

But $qv = vq'$ from the circle,

$\therefore$ $QV = VQ'$;

or $QQ'$ is bisected in $V$.

(2.) Since $NQ : Nq :: N'Q' : N'q'$

it is evident that $QQ'$ and $qq'$ will meet the axis produced in the same point.

Also the tangents to the ellipse and circle at $P$ and $p$ respectively will meet the axis in the same point.

Now in the circle the tangent at $p$ is manifestly parallel to $qq'$,

and $NQ : Nq :: mP : mp$,

$\therefore$ the tangent at $P$ is parallel to $QQ'$.

(3.) If $Cq$ be joined, since the angle $Cqo$ is a right angle and $Co$ is perpendicular to $qq'$,

$\therefore$ $Cv : Cq :: Cq : Co$, (*Euclid*, VI. 8 *Cor.*)

or, since $Cq = Cp$,

$Cv : Cp :: Cp : Co$.

$$\begin{aligned} \text{But } Cv &: Cp :: CV : CP, \\ \text{and } Cp &: Co :: CP : CO, \\ \therefore CV &: CP :: CP : CO, \\ \therefore CO &. CV = CP^2. \end{aligned}$$

**Cor.** From this it is manifest that if any number of chords be drawn parallel to each other in an ellipse, their middle points will all lie on the line drawn from the centre to the point where the tangent parallel to the chord meets the ellipse.

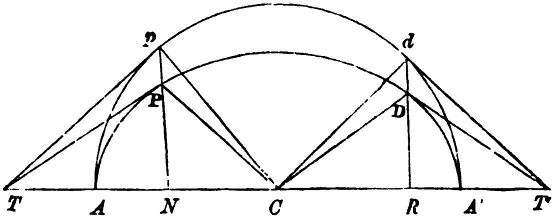
**Def.** The line $PCP'$ drawn through the centre of an ellipse and meeting the curve in $P$ and $P'$, is called a *Diameter.*

The diameter consequently bisects all chords parallel to the tangents at its extremities; and the tangents at the extremities of any chord will intersect the diameter corresponding to that chord in the same point.

36. **Def.** If $CD$ be drawn parallel to the tangent at $P$, then $CD$ is said to be *conjugate* to $CP$.

## Prop. XIX.

In the ellipse if $CD$ be conjugate to $CP$, then will $CP$ be conjugate to $CD$.



Draw the ordinates $PN$, $DR$, and produce them to meet the auxiliary circle in the points $p$, $d$.

Join $CP$, $Cp$; $CD$, $Cd$; and draw the tangents $TP$, $Tp$; $T\ D$, $T'd$.

Now, since $CD$ is parallel to $PT$,

$\therefore$ the triangle $PNT'$ is similar to the triangle $DRC$.

$\therefore$ $TN : CR :: PN : DR$,

$\therefore$ $Np : Rd$, (*Prop.* XIII. *Cor.*)

$\therefore$ $Tp$ is parallel to $Cd$,

$\therefore$ the angle $pCd$ is a right angle,

$\therefore$ $Cp$ is parallel to $T'd$,

$\therefore$ the triangle $pCN$ is similar to the triangle $dT'R$,

$\therefore$ $NC : RT' :: Np : Rd$,

$:: NP : RD$,

$\therefore$ $CP$ is parallel to $DT'$,

$\therefore$ $CP$ is conjugate to $CD$.

Cor. Since $CRd$ and $CNp$ are each similar to $dRT'$, (*Euclid*, VI. 8)

$\therefore$ the triangle $CRd$ is similar to the triangle $CNp$,

and the side $Cd$ = the side $Cp$,

$\therefore$ the triangle $CRd$ = the triangle $CNp$ in all respects,

$\therefore$ $CN = Rd$, and $CR = Np$.

Hence $DR : CN :: DR : Rd$,

$:: BC : AC$;

also $PN : CR :: PN : Np$,

$:: BC : AC$.

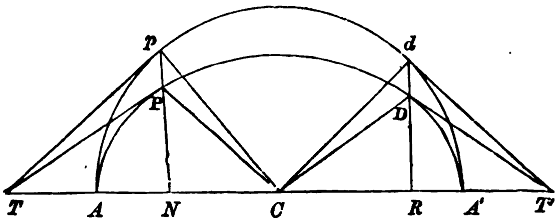
## Prop. XX.

37. If $CP$ and $CD$ be conjugate semi-diameters, and $PN$, $DR$ be the ordinates of the points $P$ and $D$; then

(1.) $CN^2 + CR^2 = AC^2$.

(2.) $PN^2 + DR^2 = BC^2$.

(3.) $CP^2 + CD^2 = AC^2 + BC^2$.



Produce $NP$, $RD$ to meet the auxiliary circle in the points $p$, $d$; then

$$CN = Rd, \quad (\textit{Prop.}\ \text{XIX.}\ \textit{Cor.})$$
$$\therefore\ CN^2 + CR^2 = Rd^2 + CR^2,$$
$$= Cd^2,$$
$$= CA^2.$$
$$\text{Again, } PN : Np :: BC : AC,$$
$$\therefore\ PN^2 : Np^2 :: BC^2 : AC^2.$$
$$\text{So } DR^2 : Rd^2 :: BC^2 : AC^2,$$
$$\therefore\ PN^2 + DR^2 : Np^2 + Rd^2 :: BC^2 : AC^2;$$
$$\text{but } Np^2 + Rd^2 = CR^2 + CN^2,$$
$$= AC^2,$$
$$\therefore\ PN^2 + DR^2 = BC^2,$$
$$\text{and } CN^2 + CR^2 = AC^2,$$
$$\therefore\ CP^2 + CD^2 = AC^2 + BC^2.$$

38. Def. A line $QV$ drawn parallel to the tangent at $P$, and meeting $CP$ in $V$, is called an *Ordinate* to the diameter $CP$.

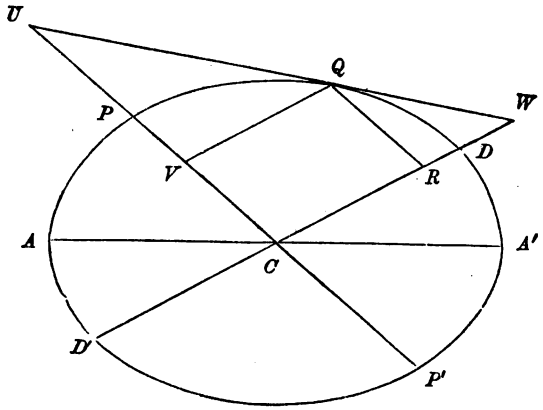
## Prop. XXI.

If $QV$ be any ordinate to the diameter $PCP'$, and $CD$ be conjugate to $CP$; then

$$QV^2 : PV \,.\, P'V :: CD^2 : CP^2.$$

Draw the tangent $UQW$ meeting $CP$ and $CD$ produced in $U$ and $W$; and draw $QR$ parallel to $CP$, meeting $CD$ in $R$.

Now, since $CR : CD :: CD : CW$, (*Prop.* XVIII.)
$\therefore CR^2 : CD^2 :: CR : CW$, (*Euclid*, VI. 20 *Cor.*)
or $QV^2 : CD^2 :: UV : CU$.



Again,

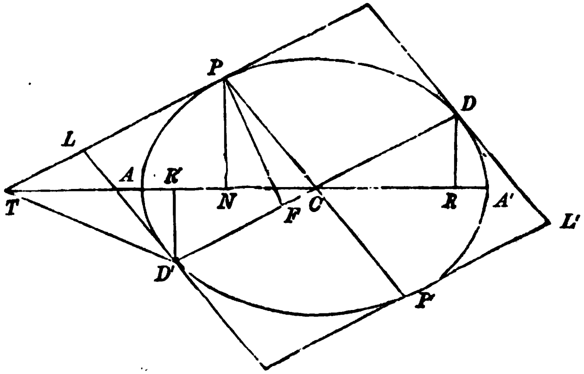
since $CU : CP :: CP : CV$, (*Prop.* XVIII.)
$\therefore CU : CV :: CP^2 : CV^2$, (*Euclid*, VI. 20 *Cor.*)
$\therefore CU - CV : CU :: CP^2 - CV^2 : CP^2$,
or $UV : CU :: PV \,.\, P'V : CP^2$.
Hence $QV^2 : CD^2 :: PV \,.\, P'V : CP^2$,
or $QV^2 : PV \,.\, P'V :: CD^2 : CP^2$.

## PROP. XXII.

39. The area of any parallelogram formed by drawing tangents to an ellipse at the extremities of a pair of conjugate

diameters is equal to the rectangle contained by the axes of the ellipse.

Let $PCP'$, $DCD'$ be a pair of conjugate diameters, and let a parallelogram be formed by drawing tangents at the points $P$, $P'$, $D$, $D'$.



Let the tangent at $P$ meet $CA$ produced in $T$; join $D'T$.

Draw the ordinates $PN$, $DR$, $D'R'$; then since $PT$ is parallel to $CD'$, the parallelogram $PD'$ is double the triangle $CTD'$, and therefore equal to the rectangle contained by $CT$ and $D'R'$.

$$\text{Now } D'R' : CN :: BC : AC, \quad (\textit{Prop. } \text{XIX. } \textit{Cor.})$$

$$\therefore CT \,.\, D'R' : CT \,.\, CN :: BC : AC, \quad (\textit{Euclid}, \text{ VI. 1.})$$

$$:: BC \,.\, AC : AC^2. \quad (\textit{Euclid}, \text{ VI. 1.})$$

$$\text{But } CT \,.\, CN = AC^2,$$

$$\therefore CT \,.\, D'R' = AC \,.\, BC,$$

$$\therefore \text{the parallelogram } LL' = 4 \text{ the parallelogram } PD',$$

$$= 4\, AC \,.\, BC,$$

$$= AA' \,.\, BB'.$$

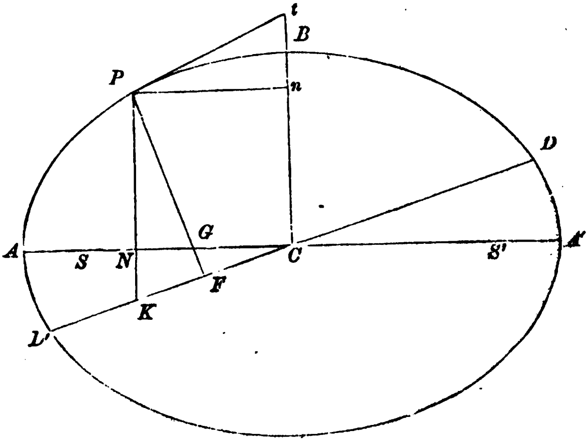
COR. If $PF$ be drawn at right angles to $DCD'$ meeting $CD'$ in $F$; then

$$PF \cdot CD' = \text{area of parallelogram } PD',$$
$$= AC \cdot BC.$$

## PROP. XXIII.

40. If $CP$ and $CD$ be conjugate diameters, and $PF$ be drawn at right angles to $CD$ meeting $CA$ in $G$, then

$$PF \cdot PG = BC^2.$$



Draw the ordinate $PN$, and produce it to meet $CD'$ in $K$.

Also draw $Pn$ at right angles to $CB$, and let the tangent at $P$ meet $CB$ produced in $t$.

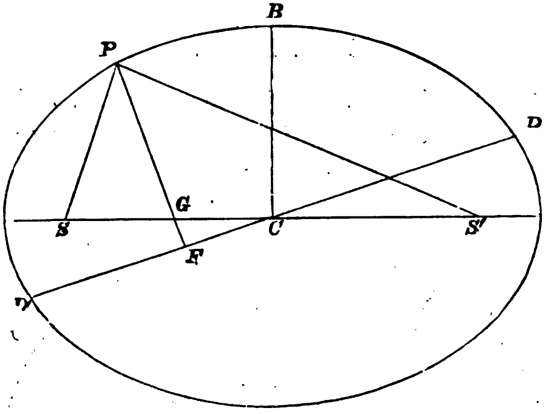
Now since the angles at $N$ and $F$ are right angles, it is evident that a circle may be described about the quadrilateral figure $NKFG$;

$$\therefore PG \cdot PF = PN \cdot PK, \quad (\textit{Euclid}, \text{III. 36 } \textit{Cor.})$$
$$= Ct \cdot Cn,$$
$$= BC^2. \quad (\textit{Prop.} \text{ XIV.})$$

### PROP. XXIV.

41. If $P$ be any point on the ellipse, and $CD$ be conjugate to $CP$, then

$$SP \,.\, S'P = CD^2.$$



Draw the normal $PG$ and produce it to meet $CD'$ in $F$; then since $CD'$ is parallel to the tangent at $P$,

$\therefore$ $PF$ is at right angles to $CD'$,

$\therefore PF \,.\, CD = AC \,.\, BC,$ (*Prop.* XXII. *Cor.*)

and $PF \,.\, PG = BC^2 = BC \,.\, BC,$ (*Prop* XXIII.)

$\therefore CD : PG :: AC : BC.$ (1)

Again, $SP : SG :: CA : CS,$ (*Prop.* XI.)

$S'P : S'G :: CA : CS.$ (*Prop.* XI.)

Compounding $SP \,.\, S'P : SG \,.\, S'G :: CA^2 : CS^2,$

$\therefore SP \,.\, S'P : SP \,.\, S'P - SG \,.\, S'G :: CA^2 : CA^2 - CS^2.$

But $SP \,.\, S'P - SG \,.\, S'G = PG^2,$ (*Euclid,* VI. *Prop.* B)

$\therefore SP \,.\, S'P : PG^2 :: CA^2 : BC^2.$

But from (1) $CD^2 : PG^2 :: CA^2 : BC$

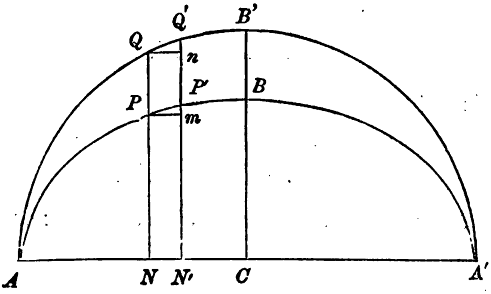
$\therefore SP \,.\, S'P = CD^2.$

This proposition may also be very easily deduced from *Prop.* XV.

### PROP. XXV.

take

42. The area of the ellipse is to the area of the auxiliary circle as $BC$ to $AC$.



Let $PN$ and $P'N'$ be two ordinates of the ellipse near together.

Produce $NP$, $N'P'$, to meet the auxiliary circle in $Q$ and $Q'$.

Draw $Pm$, $Qn$, perpendicular to $Q'N'$.

Then

the parallelogram $PN'$ : the parallelogram $QN'$ :: $PN : QN$,
:: $BC : AC$.

And the same will be true for all the parallelograms that can be similarly described in the ellipse and auxiliary circle.

Hence the sum of all the parallelograms inscribed in the ellipse is to the sum of all the parallelograms inscribed in the circle as $BC$ to $AC$.

And this holds however the number of parallelograms be increased.

But when the number of parallelograms is increased, and the breadth of each diminished indefinitely, the sum of the parallelograms inscribed in the ellipse will be equal to the area of the ellipse, and the sum of those inscribed in the circle to the area of the circle. Hence

the area of the ellipse : the area of the circle :: $BC : AC$.

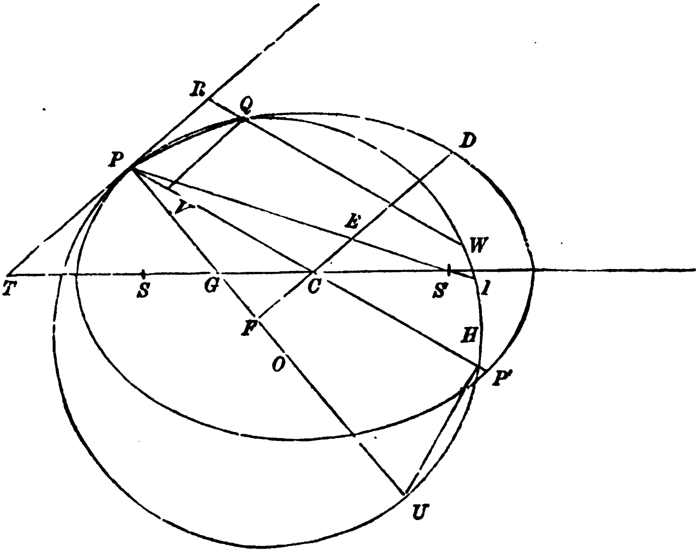
43. Def. If with a point $O$ on the normal at $P$ as centre, and $OP$ as radius, a circle be described touching the ellipse at $P$, and cutting it in $Q$; then when the point $Q$ is made to approach indefinitely near to $P$, the circle is called the *Circle of Curvature* at the point $P$.

## Prop. XXVI.

If $PH$ be the chord of the circle of curvature at the point $P$ of an ellipse, which passes through the centre; then

$$PH \cdot CP = 2\, CD^2.$$

Let $PT$ be the tangent, and $PG$ the normal at the point $P$.



With centre $O$, and radius $OP$, describe a circle cutting the ellipse in the point $Q$.

Draw $RQW$ parallel to $CP$, meeting the circle in $W$, and $TP$ produced in $R$.

Also draw $QV$ parallel to $PR$, meeting the diameter $PP'$ in $V$; then since $RP$ touches the circle at $P$,

$$\therefore RQ \,.\, RW = PR^2, \quad (\textit{Euclid}, \text{III. } 36)$$

$$\text{or } PV \,.\, RW = QV^2.$$

$$\text{But } QV^2 : PV \,.\, P'V :: CD^2 : CP^2, \quad (\textit{Prop.} \text{ XXI.})$$

$$\therefore PV \,.\, RW : PV \,.\, P'V :: CD^2 : CP^2,$$

$$\text{or } RW : P'V :: CD^2 : CP^2.$$

Now, when the circle becomes the circle of curvature at $P$, the points $R$ and $Q$ move up to, and coincide with $P$, and the lines $RW$ and $PH$ become equal, while

$$P'V \text{ becomes equal to } PP' \text{ or } 2CP.$$

$$\text{Hence, } PH : 2CP :: CD^2 : CP^2,$$

$$\therefore PH \,.\, CP : 2CP^2 :: 2CD^2 : 2CP^2,$$

$$\therefore PH \,.\, CP = 2CD^2.$$

### PROP. XXVII.

If $PU$ be the diameter of the circle of curvature at the point $P$ of the ellipse, and $PF$ be drawn at right angles to $CD$; then

$$PU \,.\, PF = 2CD^2.$$

Since the triangle $PHU$ is similar to the triangle $PFC$,

$$\therefore PU : PH :: CP : PF,$$

$$\therefore PU \,.\, PF = PH \,.\, CP,$$

$$= 2CD^2. \quad (\textit{Prop.} \text{ XXVI.})$$

### PROP. XXVIII.

If $PI$ be the chord of the circle of curvature through the focus of the ellipse; then

$$PI \,.\, AC = 2CD^2.$$

Let $PI$ meet $CD$ in $E$; then, since the triangles $PIU$ and $PEF$ are similar,

$$\therefore PI : PU :: PF : PE.$$

But $PE = AC$, (*Prop.* XV. *Cor.*)

$$\therefore PI : PU :: PF : AC,$$

$$\therefore PI \, . \, AC = PU \, . \, PF,$$

$$= 2\,CD^2. \quad (\textit{Prop. } \text{XXVII.})$$

### Prop. XXIX.

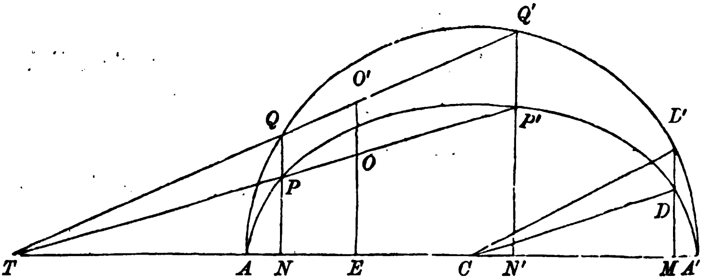
44. If two chords of an ellipse intersect one another, the rectangles contained by their segments are proportional to the squares of the diameters parallel to them.

Let $POP'$ be any chord drawn through the point $O$, and let $CD$ be the semi-diameter parallel to it.

Draw the ordinates $NP$, $N'P'$, $MD$, and produce them to meet the auxiliary circle in $Q$, $Q'$, $D'$; then

since $NP : NQ :: N'P' : N'Q'$, (*Prop.* XIII. *Cor.*)

it is evident that $PP'$ and $QQ'$ will meet the axis produced in the same point $T$.



Also since $NP : NQ :: MD : MD'$, (*Prop.* XIII. *Cor.*)

and $TPP'$ is parallel to $CD$,

$\therefore TQQ'$ is parallel to $CD'$.

Draw $EO$ parallel to $NQ$ or $N'Q'$, and produce it to meet $QQ'$ in $O'$; then

$$\begin{aligned} PO : QO' &:: TO : TO', \\ \text{and } P'O : Q'O' &:: TO : TO', \\ \therefore PO \,.\, P'O : QO' \,.\, Q'O' &:: TO^2 : TO'^2, \\ &:: CD^2 : CD'^2, \\ &:: CD^2 : AC^2. \end{aligned}$$

Alternately, $PO \,.\, P'O : CD^2 :: QO' \,.\, Q'O' : AC^2$.

Again, if through the point $O$ any other chord $pOp'$ be drawn,

$$\text{since } EO : EO' :: BC : AC,$$

it is manifest that the corresponding chord $qq'$ in the auxiliary circle will pass through the point $O'$; and if $Cd$ be the semi-diameter parallel to $pp'$ we shall have as before,

$$pO \,.\, p'O : Cd^2 :: qO' : q'O' : AC^2.$$

But $QO' \,.\, Q'O' = qO' \,.\, q'O'$, (*Euclid*, III. 35.)

$$\begin{aligned} PO \,.\, P'O &: CD^2 :: pO \,.\, p'O : Cd^2, \\ PO \,.\, P'O &: pO \,.\, p'O :: CD^2 : Cd^2. \end{aligned}$$

The same result may be shown to be true when the point $O$ is without the ellipse.

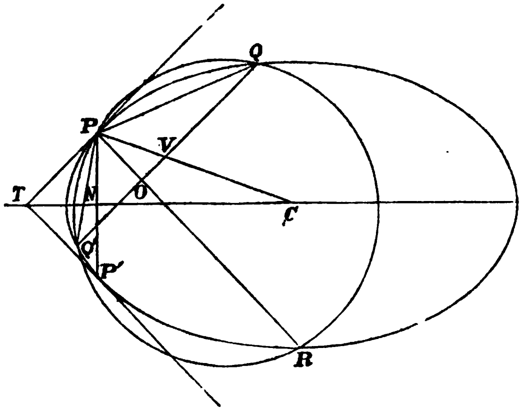
## PROP. XXX.

If $QVQ'$ be any ordinate to the diameter $CP$, the circle described through the three points $P$, $Q$, $Q'$ will intersect the ellipse in a fourth point, which depends only upon the position of $P$.

Draw the ordinate $PN$, and produce it to meet the ellipse in $P'$; then, since, if $NT$ be the subtangent of either $P$ or $P'$,

$$CT \,.\, CN = AC^2, \text{ (}Prop.\text{ IX.)}$$

therefore the tangents at $P$ and $P'$ will meet the major axis produced in the same point $T$.



Draw $PR$ parallel to $TP'$, meeting the ellipse in $R$, and $QQ'$ in $O$; then if $CD$ and $CD'$ be drawn parallel respectively to $TP$ and $TP'$, meeting the ellipse in $D$ and $D'$,

$$PO \cdot OR : QO \cdot OQ' :: CD'^2 : CD^2. \quad (\textit{Prop.}\ \text{XXIX.})$$

$$\text{But } CD' = CD, \text{ since } CP' = CP,$$

$$\therefore PO \cdot OR = QO \cdot OQ'.$$

Hence, by the converse of *Euclid*, III. *Prop.* 35, the point $R$ is on the circle which passes through $P$, $Q$, $Q'$.

Cor. When the point $V$ is brought indefinitely near to $P$, $QQ'$ coincides with the tangent to the ellipse at $P$, and becomes also a tangent to the circle at $P$ since $Q$ and $Q'$ are indefinitely near to each other. The circle therefore becomes the circle of curvature at the point $P$.

Hence, if $PR$ be drawn parallel to the tangent at $P'$, or be equally inclined to the axis with $PT$, it will meet the ellipse in the point where the circle of curvature at $P$ intersects the ellipse.

# PROBLEMS ON THE ELLIPSE.

1. In what position of $P$ is the angle $SPS'$ greatest?

2. The latus rectum is a third proportional to the axis major and axis minor.

3. Construct on the axis minor as base, a rectangle which shall be to the triangle $SLS'$ in the duplicate ratio of the major axis to the minor axis, $L$ being the extremity of the latus rectum.

4. If a series of ellipses be described having the same major axis; the tangents at the extremities of their latera recta will all meet the minor axis in the same point.

5. Find the locus of the centres of all the ellipses having the same focus, and their major axes of the same length, and touching a given straight line.

6. Given the foci, it is required to describe an ellipse touching a given straight line.

7. If $PT$ be a tangent to an ellipse, meeting the axis in $T$, and $AP$, $A'P$, be produced to meet the perpendicular to the major axis through $T$ in $Q$ and $Q'$, then $QT = Q'T$.

8. If the angle $SBS'$ be a right angle, prove that $CA^2 = 2CB^2$.

9. If $CP$ be a semi-diameter, and $AQO$ be drawn parallel to $CP$ meeting the curve in $Q$, and $CB$ produced in $O$, then $2CP^2 = AO \cdot AQ$.

10. If $AB$, $CD$, which are not parallel, make equal angles with either axis, the lines $AC$, $BD$, as also $AD$, $BC$, will make equal angles with either axis.

11. $PSp$ is any focal chord. $PA$ and $pA$ are produced to meet the directrix in $Q$ and $q$. Prove that the angle $QSq$ is a right angle.

12. If a circle be described touching the axis major in one focus, and passing through one extremity of the axis minor; $AC$ will be a mean proportional between the diameter of this circle and $BC$.

13. If $PQQ'P'$ be a chord of the auxiliary circle, and a circle be described on the minor axis as diameter, cutting the chord in $Q$ and $Q'$, then $PQ \,.\, P'Q = CS^2$.

14. If $PG$ be the normal at $P$, and $GL$ be drawn at right angles to $SP$, then $PL = \frac{1}{2}$ latus rectum.

15. The sum of the squares of the normals at the extremities of conjugate diameters is constant.

16. If on the normal at $P$, $PQ$ be taken equal to the semi-conjugate diameter $CD$, the locus of $Q$ is a circle whose radius is $AC - BC$.

17. Find the locus of the intersection of a pair of tangents at right angles to each other.

18. $P$ is any point on an ellipse. To any point $Q$ on the curve draw $AQ$, $A'Q$, meeting $NP$ in $R$ and $S$, and prove that $NR \,.\, NS = NP^2$.

19. If $PG$ be a normal, and $GL$ perpendicular to $SP$, the ratio of $GL$ to $PN$ is constant.

20. If $NP$ produced meet the tangent at the extremity of the latus rectum in $Q$, then $QN = PS$.

21. In an ellipse the tangent at any point makes a greater angle with the focal distance than with the perpendicular on the directrix.

22. A diameter of an ellipse, parallel to the tangent at any point, meets the focal distances of the point, and from the points of intersection lines are drawn perpendicular to the focal distances. Prove that these lines intersect in the axis minor.

23. The subnormal is a third proportional to $CT$ and $BC$.

24. If $PN$ be the ordinate of $P$, prove that $NY : NY' :: PY : PY'$. (*See fig. Prop.* XV.)

25. If from $C$ lines be drawn parallel and perpendicular to the tangent at $P$, they inclose a part of one of the focal distances of that point equal to the other.

26. If $P$ be a fixed point on an ellipse, and $QQ'$ an ordinate to $CP$, the circle $QPQ'$ will meet the ellipse in a fixed point.

27. $P$ is any point on an ellipse. Draw $PP'$ parallel to the axis major, and through $P'$ draw $P'Q$, $P'Q'$, making equal angles with the major axis. Join $QQ'$; then $QQ'$ is parallel to the tangent at $P$.

28. What parallelogram circumscribing an ellipse has the least area?

29. When is the square of the sum of conjugate diameters least?

30. Given the axes of an ellipse, and the position of one focus, and of one point in the curve, give a geometrical construction for finding the centre.

31. If lines drawn through any point of an ellipse to the extremities of any diameter meet the conjugate $CD$ in $M$ and $N$, then $CM . CN = CD^2$.

32. If $CP$ and $CD$ be conjugate, prove that

$$(SP - AC)^2 + (SD - AC)^2 = SC^2.$$

33. If $CP$ and $CD$ be conjugate, and $BP$, $PD$ be joined, as also $AD$, $A'P$, these latter meeting in $O$, then $BDOP$ is a parallelogram. When is the area greatest?

34. If $PSp$, $QCq$ be two parallel chords through the focus and centre of an ellipse, prove that

$$SP . Sp : CQ . Cq :: BC^2 : AC^2.$$

35. If the tangent at the vertex $A$ cut any two conjugate diameters in $T$ and $t$, then $AT . At = BC^2$.

36. If the tangents at three points $P$, $Q$, $R$, intersect in $R_{,}$, $Q_{,}$, $P_{,}$, prove that

$$PR_{,} . P_{,}Q . Q_{,}R = PQ_{,} . R_{,}Q . P_{,}R.$$

37. If a circle be described touching $SP$, $S'P$ produced, and the major axis of the ellipse, find the locus of the centre.

38. If from the extremities of the axes of an ellipse any four parallel lines be drawn, the points in which they cut the curve are the extremities of conjugate diameters.

39. If two equal and similar ellipses have a common centre, the points of intersection are at the extremities of diameters at right angles to one another.

40. If $PSQ$ be a focal chord, and $X$ the foot of the directrix, $XP$ and $XQ$ are equally inclined to the axis.

41. $OP$, $OQ$ are tangents to an ellipse, and $PQ$ is produced to meet the directrices in $R$, $R'$, prove that

$$RP \,.\, R'P : RQ : R'Q :: OP^2 : OQ^2.$$

42. $NPQ$ is a common ordinate to the ellipse and auxiliary circle. $PR$, $QR$ are normals at $P$ and $Q$ intersecting in $R$. The locus of $R$ is a circle whose radius is $AC + BC$.

43. If the conjugate to $CP$ meet $SP$, $S'P$, or these produced in $E$, $E'$; then $SE = S'E'$, and the circles circumscribing $SCE$, $S'CE'$ are equal.

44. The locus of the middle points of all focal chords in an ellipse is a similar ellipse.

45. The circle described about the triangle $SBS'$ will cut the minor axis in the centre of the circle of curvature at $B$.

46. The locus of the centre of the circle inscribed in the triangle $SPS'$ is an ellipse.

47. If a circle be described intersecting an ellipse in four points, and chords be drawn through the points of intersection, diameters parallel to the chords will be equal.

48. An ellipse slides between two lines at right angles to each other, find the locus of its centre.

49. If from the focus $S$ perpendiculars be drawn upon the conjugate diameters $CP$, $CD$, these perpendiculars produced backward will intersect $CD$ and $CP$ in the directrix.

50. Find the point at which the diameter of curvature is a mean proportional between the major and minor axes.

51. The circle of curvature at a point, where the conjugate diameters are equal, meets the ellipse again at the extremity of the diameter.

52. The locus of the intersection of lines drawn from $A$, $A$ at right angles to $AP$, $A'P$ is an ellipse.

53. If a quadrilateral figure be inscribable in two ellipses whose major axes are parallel or perpendicular, any two of its opposite angles will be equal to two right angles.

54. If $CN$, $NP$ are the abscissa and ordinate of a point $P$ on a circle whose centre is $C$, and $NQ$ be taken equal to $NP$, and be inclined to it at a constant angle, the locus of $Q$ is an ellipse.

55. If two ellipses having the same major axes can be inscribed in a parallelogram, the foci will be on the corners of an equiangular parallelogram.

56. Two ellipses, whose major axes are equal, have a common focus. Prove that they intersect in two points only.

57. A circle described about the triangle $SPS'$ cuts the minor axes in $R$ on the opposite side to $P$. Prove that $SR$ varies as the normal $PG$.

58. If $r$ and $R$ be the radii of the circles inscribed in and about the triangle $SPS$, prove that $R . r$ varies as $SP . S'P$.

59. The circle described upon $PG$ as diameter cuts $SP$, $S'P$ in $K$ and $L$. Prove that $KL$ is bisected by $PG$, and is perpendicular to it.

60. If from $S'$ a line be drawn parallel to $SP$, it will meet $SY$ in the circumference of a circle.

61. $T$ and $t$ are the points where the tangent at $P$ meets the axes. $CP$ is produced to meet in $L$ the circle described about the triangle $TCt$; prove that $PL$ is half the chord of the circle of curvature at $P$ in the direction of $C$, and that $CP . CL$ is constant.

62. About the triangle $PQR$ an ellipse is described, having its centre at the point where the lines drawn from $P$, $Q$, $R$, to the middle points of the opposite sides meet. $CP$, $CQ$, $CR$, are produced to meet the ellipse in $P'$, $Q'$, $R'$. Prove that

the tangents at $P'$, $Q'$, $R'$ form a triangle similar to $PQR$, and four times as large.

63. Lines from $Y$ and $Y'$ perpendicular to the major axis cut the circles on $SP$, $S'P$ as diameters in $I$ and $J$. Prove that $IS$ and $JS'$ when produced, intersect $BC$ in the same point.

64. If from the ends of any diameter chords be drawn to any point in the ellipse, the diameters parallel to these chords will be conjugate.

65. If $T$ be the angle between tangents at the extremities of a focal chord, and $O$ the angle subtended by the chord at the other focus, then

$$2T + O = 2 \text{ right angles.}$$

66. The acute angles which $SP$, $SQ$ make with the tangents are complementary. Prove that $BC^2$ is a mean proportional between the areas of the triangles $SPS'$, $SQS'$. Also, show that the problem is impossible unless $BC < CS$.

67. A series of ellipses have their equal conjugate diameters of the same magnitude. One of these diameters is fixed and common, while the other varies. The tangents drawn from any point in the fixed diameter produced will touch the ellipses in points situated on a circle.

68. If on the longer side of a rectangle as major axis an ellipse be described, passing through the intersection of the diagonals, and lines be drawn from any point of the ellipse exterior to the rectangle to the ends of the remote side, they will divide the major axis into segments, which are in geometric progression.

69. From any point $P$ of an ellipse $PQ$ is drawn at right angles to $SP$ meeting the diameter conjugate to $CP$ in $Q$. Prove that $PQ$ varies inversely as the perpendicular from $P$ on the major axis.

70. In an ellipse $SQ$ and $S'Q$, drawn at right angles to a pair of conjugate diameters, intersect in $Q$. Prove that the locus of $Q$ is a concentric ellipse.

# CHAPTER III.

## THE HYPERBOLA.

45. Def. The *Hyperbola* is the curve traced out by a point which moves in such a manner that its distance from a given fixed point continually bears the same ratio, *greater than unity*, to its distance from a given fixed line. (*See Introduction.*)

### Prop. I.

The focus and directrix of a hyperbola being given, to find any number of points on the curve.

Let $S$ be the focus, and $MX$ the directrix.

Draw $SX$ at right angles to the directrix, and divide $SX$ in the point $A$, so that $SA$ may be to $AX$ in the given fixed ratio, greater than unity; then

$A$ is a point on the curve.

On $SX$ produced take a point $A'$, such that

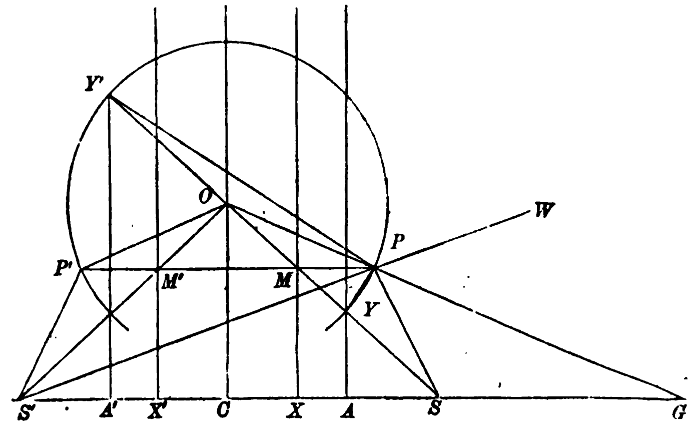
$$SA' : A'X :: SA : AX;$$

then $A'$ will also be a point on the curve.

On the directrix take *any* point $M$; and through $S$ and $M$ draw the line $SYMY'$, meeting $AY$ and $A'Y'$, drawn at right angles to $AA'$, in the points $Y$ and $Y'$;

On $YY'$ as diameter describe a circle, and draw $PMP$ parallel to $AA'$, cutting the circle in the points $P$ and $P'$;

$P$ and $P'$ will be points on the hyperbola.



Join $PY$, $PY'$, $SP$; then since

$SY : YM :: SA : AX$, (*Euclid*, VI. 2)

and $SY' : Y'M :: SA' : A'X$, (*Euclid*, VI. 2)

$\therefore SY : YM :: SY' : Y'M$;

or, alternately, $SY : SY' :: YM : Y'M$,

and the angle $YPY'$ in a semicircle is a right angle,

$\therefore PY$ bisects the angle $SPM$,*

$\therefore SP : PM :: SY : YM$,

$:: SA : AX$.

So we may show that

$SP' : P'M :: SY' : Y'M$,

---

* For, if not, make the angle $YPm$ equal to $YPS$; then

$SY : Ym :: SP : Pm$. (*Euclid*, VI. 1.)

and since, if $PY$ bisect $SPm$, $PY'$ being at right angles to $PY$, also bisects the angle between $MP$ and $SP$ produced;

$\therefore SY' : Y'm :: SP : Pm$, (*Euclid*, VI. A.)

Hence $SY : Ym :: SY' : Y'm$,

or $SY : SY' :: Ym : Y'm$,

$\therefore$ the points $M$ and $m$ coincide.

$$:: SA : AX,$$

$$\therefore P \text{ and } P' \text{ are points on the curve.}$$

In the same way, by taking other points on the directrix, we may obtain as many more points on the curve as we please.

Cor. 1. Since, corresponding to every point $P$ on the curve, there is a point $P'$ situated in precisely the same manner with respect to $A'Y'$ as $P$ is with respect to $AY$, it is clear that if we make $A'S'$ equal to $AS$, and $A'X'$ equal to $AX$, and draw $X'M'$ at right angles to $AX'$, the curve could be equally well described with $S'$ as focus and $M'X'$ as directrix.

The hyperbola is therefore symmetrical, not only with respect to the line $AA'$, but also with respect to the line $OC$ drawn through the middle point of $YY'$ at right angles to and bisecting $AA'$.

Cor. 2. The line $OP$ produced will bisect the angle $SPW$ between $SP$ and $S'P$ produced.

Produce $OP$ and $S'S$ to meet in $G$. Produce $PM$ to meet $X'M'$ in $M'$, and draw $OS'$ passing through the point $M'$; then

$$SP : PM :: S'P : PM',$$

$$\text{or, alternately, } SP : S'P :: PM : PM'. \quad (1)$$

$$\text{Again, } SG : PM :: S'G : PM',$$

$$\text{or, alternately, } SG : S'G :: PM : PM'. \quad (2)$$

$$\therefore \text{ from (1) and (2)}$$

$$SP : S'P :: SG : S'G,$$

$$\therefore PG \text{ bisects the angle } SPW. \quad (\textit{Euclid}, \text{VI. A.})$$

It will be shown hereafter (*Prop.* IX.) that the normal to the hyperbola at the point $P$ also bisects the angle $SPW$. Hence the hyperbola and circle have the same tangent at the point $P$. The hyperbola will consequently touch all the infinite series of circles which can be described in the same manner as the one in the figure, by taking different points on the directrix.

## PROP. II.

46. If $C$ be the middle point of $AA'$, then $CA$ is a mean proportional between $CS$ and $CX$,

$$\text{or } CS \,.\, CX = CA^2. \quad (\textit{See fig. Prop.}\ \text{III.})$$

$$\text{Since } SA' : A'X :: SA : AX.$$

$$\text{Alternately } SA' : SA :: A'X : AX,$$

$$\therefore SA' - SA : SA :: A'X - AX : AX;$$

$$\text{or } AA' : SA :: XX' : AX,$$

$$\therefore AA' : XX' :: SA : AX,$$

$$\text{or } CA : CX :: SA : AX. \quad (1.)^*$$

$$\text{Again, } SA' : SA :: A'X : AX.$$

$$\therefore SA' + SA : SA :: A'X + AX : AX,$$

$$\text{or } SS' : SA :: AA' : AX.$$

$$\text{Alternately, } SS' : AA' :: SA : AX,$$

$$\text{or } CS : CA :: SA : AX. \quad (2.)$$

Hence from (1) and (2)

$$CA : CX :: CS : CA,$$

$$\therefore CA^2 = CX \,.\, CS.$$

or $CA$ is a mean proportional between $CS$ and $CX$.

COR. Since the three lines $CS$, $CA$, $CX$, are proportional therefore, by the definition of duplicate ratio and *Euclid*. VI. 20 *Cor.*,

$$CS : CX :: CS^2 : CA^2. \quad (3.)$$

## PROP. III.

47. If $P$ be any point on the hyperbola, and $S$ be the focus nearer to $P$; then

$$S'P - SP = AA'.$$

$$\text{Since } SP : PM :: SA : AX,$$

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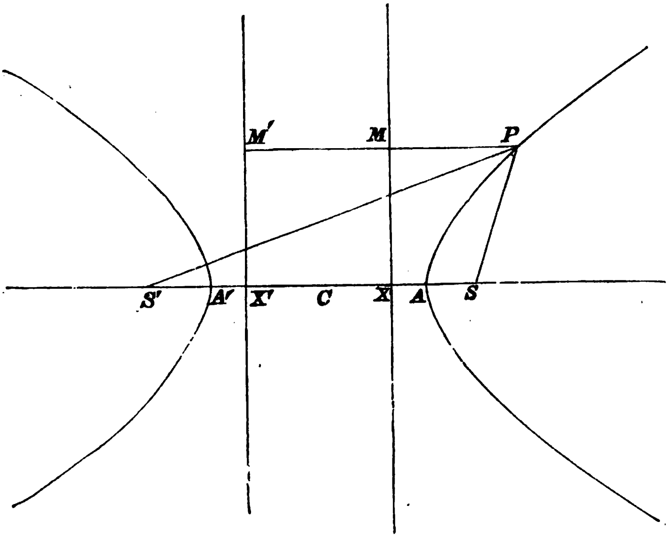
[*] N.B. The results (1), (2), (3), should be remembered, as they will frequently be referred to.

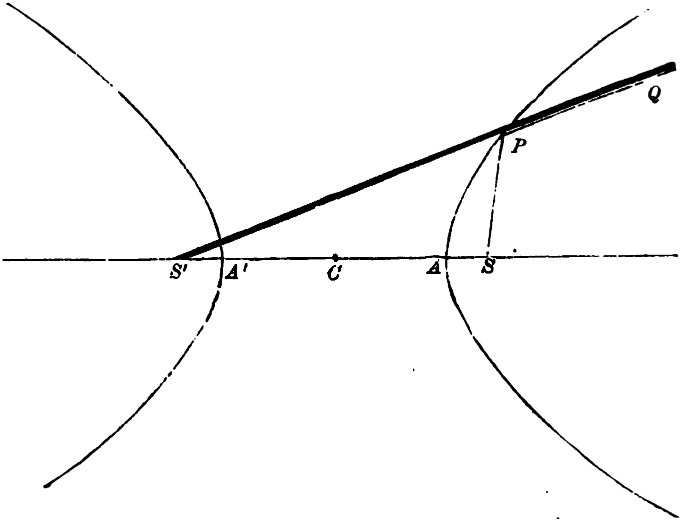


$$\text{and } SA : AX :: AA' : XX', \quad (\textit{Prop. } \text{II.})$$
$$\therefore\ SP : PM :: AA' : XX'.$$
$$\text{So } S'P : PM' :: AA' : XX',$$
$$\therefore\ S'P - SP : PM' - PM :: AA' : XX'.$$
$$\text{But } PM' - PM = MM' = XX',$$
$$\therefore\ S'P - SP = AA'.$$

COR. By means of this property the hyperbola may be practically described, and the form of the curve determined.

Let a rigid bar $S'Q$ of any length have one end fastened at the focus $S'$, in such a manner that it is capable of turning freely round $S'$ as a centre in the plane of the paper.

At the other end of the bar let a string be fastened of such a length that when stretched along the bar it shall just reach to within a distance equal to $AA'$ from the end $S'$ of the bar.



If the loose end of the string be now fastened to the focus $S$, and the rod being initially placed in the position $S'S$, be made to revolve round $S'$, while the string is kept constantly stretched by means of the point of a pencil at $P$, in contact with the bar; since $S'P$ and $SP$ are always increasing by the same amount, viz. the length of the portion of the string that removes itself from the bar, between any two positions of $P$, the difference between $S'P$ and $SP$ will be constantly the same, and the point $P$ will trace out the hyperbola.

Another perfectly similar branch may be described in the same manner by making the bar revolve round $S$ as centre.

In this case $S'P - SP$ will be equal to $AA'$.

The curve, therefore, consists of two similar branches, which recede indefinitely both from the line $AA'$, and also from the line $BCB'$ drawn bisecting $AA'$ at right angles. (*See fig. Prop.* IV.)

48. If $BC$ be taken of such a length that

$$BC^2 = CS^2 - CA^2,$$

and $CB'$ be made equal to $CB$, then $AA'$ and $BB'$ are called respectively the *Transverse* and *Conjugate Axes.*

The line $BCB'$ does not meet the hyperbola, and the reason of its being introduced will be seen further on.

If the conjugate and transverse axes are equal, the hyperbola is said to be *rectangular* or *equilateral.*

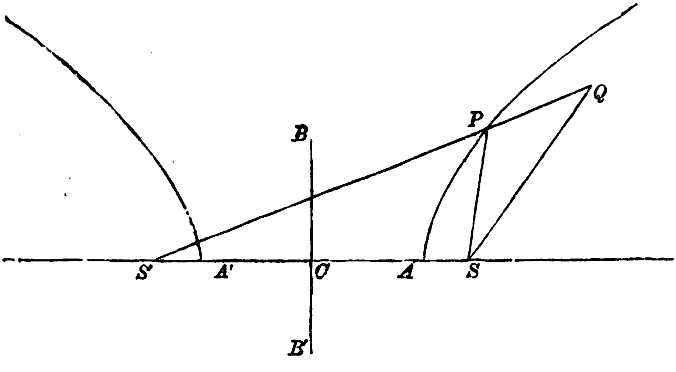
The property of the hyperbola, which we have just investigated, viz. that the difference between $SP$ and $S'P$ is constant, is sometimes taken as the definition of the curve. (*See Chapter* II. *Art.* 25.)

Also as in the ellipse, if $SL$ be the semi-latus rectum, it may be proved that

$$SL \,.\, AC = BC^2.$$

### PROP. IV.

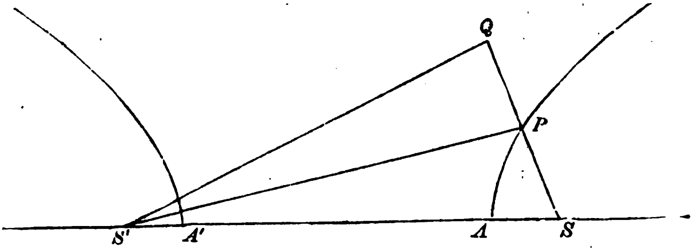
49. The difference of the distances of any point from the foci of a hyperbola will be greater or less than $AA'$, according as the point is on the concave or convex side of the curve.



(1.) Let $Q$ be a point on the concave side of the hyperbola.

Join $SQ$, $S'Q$, and let $S'Q$ meet the curve in $P$; join $SP$; then

$$\text{since } S'Q = S'P + PQ,$$
$$\text{and } SQ < SP + PQ,$$



$$\therefore\ S'Q - SQ > S'P - SP.$$
$$\text{but } S'P - SP = AA',$$
$$\therefore\ S'Q - SQ > AA'.$$

(2.) Let $Q$ be a point on the convex side of the curve nearer to $S$ than $S'$; join $SQ$, $S'Q$, and let $SQ$ meet the curve in $P$; join $S'P$; then

$$S'Q < S'P + PQ,$$
$$\text{and } SQ = SP + PQ,$$
$$\therefore\ S'Q - SQ < S'P - SP,$$
$$\text{but } S'P - SP = AA'.$$
$$\therefore\ S'Q - SQ < AA',$$

so if $Q$ be nearer to $S'$ than $S$, we can show that

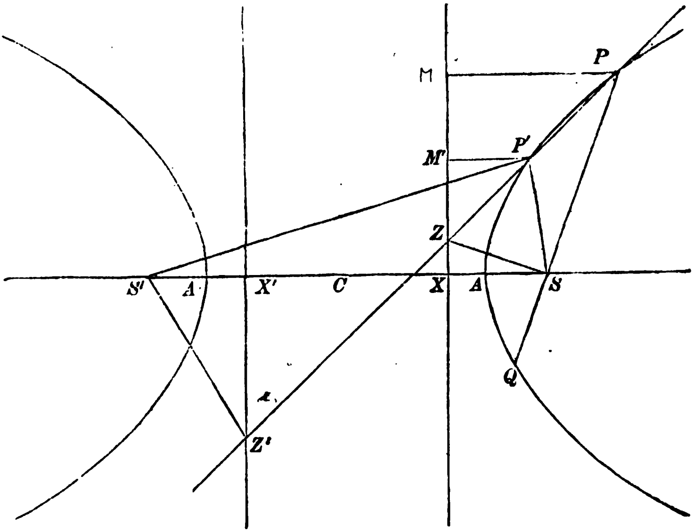
$$SQ - S'Q < AA';$$

Cor. Conversely a point will be on the concave or convex side of the hyperbola, according as the difference of its distances from the foci is greater or less than $AA'$.

50. Def. If a point $P'$ be taken on the hyperbola near to $P$ (*see fig. Prop.* V.) and $PP'$ be joined, the line $PP'$ produced, in the limiting position which it assumes when $P'$ is made to approach indefinitely near to $P$, is called the *Tangent* to the hyperbola at the point $P$.

## Prop. V.

If the tangent to the hyperbola at any point $P$ meet the directrix in the point $Z$, and if $S$ be the focus corresponding to the directrix on which $Z$ is situated, then $SZ$ will be at right angles to $SP$.



Let $P'$ be a point in the curve near to $P$.

Draw the chord $PP'$, and produce it to meet the directrix in $Z$; join $SZ$.

Draw $PM$, $P'M'$ at right angles to the directrix, and join $SP$, $SP'$.

Produce $PS$ to meet the hyperbola in $Q$; then since the triangles $ZMP$, $ZM'P'$ are similar,

$$\therefore ZP : ZP' :: MP : M'P',$$
$$:: SP : SP'.$$

$\therefore$ $SZ$ bisects the angle $P'SQ$. (*Euclid*, VI. A.)

Now, when $P'$ is indefinitely near to $P$, and $PP'$ becomes the tangent at the point $P$, the angle $PSP'$ becomes indefinitely small, while the angle $QSP'$ approaches two right angles; and therefore the angles $ZSP'$, being half of the angle $P'SQ$, becomes ultimately a right angle.

Hence, when $PZ$ becomes the tangent at the point $P$, the angle $ZSP$ is a right angle,

or $SZ$ is perpendicular to $SP$.

Cor. 1. Conversely, if $SZ$ be drawn at right angles to $SP$, meeting the directrix in $Z$, and $PZ$ be joined, $PZ$ will be the tangent at $P$.

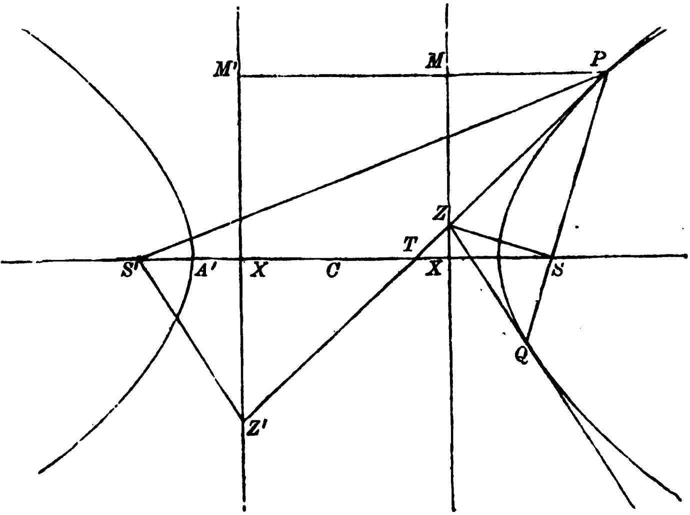
Cor. 2. If $PZ$ be produced to meet the other directrix in $Z'$, and $S'Z'$ be joined; then

$$S'Z' \text{ is at right angles to } S'P'.$$

Cor. 3. The tangents at the extremities of the latus rectum, or double ordinate through the focus, meet the axis in the point $X$.

## Prop. VI.

The tangent to the hyperbola at any point $P$ makes equal angles with the focal distances $SP$ and $S'P$.



Let the tangent at $P$ meet the directrices in $Z$ and $Z'$.

Draw $PMM'$ at right angles to the directrices meeting them in $M$ and $M'$ respectively; join $SZ$, $S'Z'$; then

$$SP : PM :: S'P : PM'.$$

And since the triangles $ZMP$, $Z'M'P$ are similar,

$$PM : PZ :: PM' : PZ',$$

$$\therefore\ SP : PZ :: S'P : PZ'.\quad (\textit{Ex æquali.})$$

Now in the triangles $SPZ$, $S'PZ'$ because the sides about the angles $SPZ$, $S'PZ'$ are proportional, and the angles $PSZ$, $PS'Z'$ are equal, being right angles, and the angles $SZP$, $S'Z'P$ are each less than a right angle,

$\therefore$ the triangles $SPZ$, $S'PZ'$ are similar. (*Euclid*, VI. 7).

$\therefore$ the angle $SPZ = S'PZ'$.

### Prop. VII.

Omit

The tangents at the extremities of a focal chord intersect in the directrix.

Let $PSQ$ be a focal chord, and let the tangent $P$ meet the directrix in $Z$. Join $SZ$; then

the angle $ZSP$ is a right angle, (*Prop.* V.)

And $\therefore$ also the angle $ZSQ$ is a right angle,

$\therefore$ $ZQ$ is the tangent at $Q$. (*Prop.* V. *Cor.* 1.)

Or the tangents at the extremities of a focal chord intersect in the directrix.

### Prop. VIII.

51. If the tangent at $P$ meet the transverse axis in $T$, and $PN$ be the ordinate of the point $P$; then

$$CT \,.\, CN = CA^2.$$

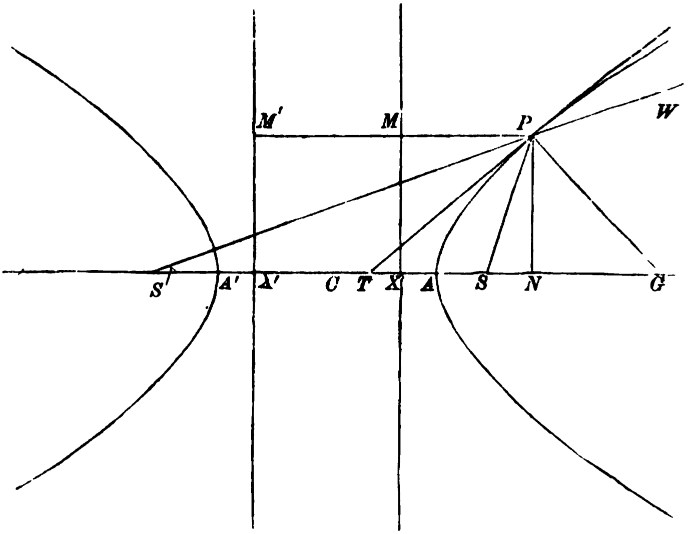
Draw $PMM'$ at right angles to the directrices meeting them in $M$ and $M'$. Join $SP$, $S'P$; then

since $PT$ bisects the angle $SPS'$, (*Prop.* VI.)

$$\therefore\ S'T : ST :: S'P : SP,\ (\textit{Euclid}, \text{VI. } 3.)$$

$$:: PM' : PM,$$

$$:: X'N : XN.$$



$$\therefore S'T - ST :: S'T + ST :: X'N - XN : X'N + XN$$

$$\text{or } 2CT : 2CS :: 2CX : 2CN,$$

$$\text{or } CT : CS :: CX : CN.$$

$$\therefore CT \,.\, CN = CS \,.\, CX,$$

$$= CA^2. \quad (\textit{Prop.}\ \text{II.})$$

52. Def. The line $PG$, drawn at right angles to the tangent $PT$, is called the *Normal* to the hyperbola at the point $P$.

### Prop. IX.

If the normal to the hyperbola at the point $P$ meet the transverse axis in the point $G$, and $PN$ be the ordinate of the point $P$, then

$$NG : NC :: BC^2 : AC^2.$$

Draw $PMM'$ at right angles to the directrices, meeting them in $M$ and $M'$, and produce $S'P$ to $W$; then since

the angle $TPG$ is a right angle,

∴ the angle $WPG$ = the complement of the angle $S'PT$,

and the angle $SPG$ = the complement of the angle $SPT$;

but the angle $S'PT$ = the angle $SPT$,

$\therefore$ the angle $WPG$ = the angle $SPG$,

$\therefore$ $PG$ bisects the angle $SPW$,

$$\therefore S'G : SG :: S'P : SP, \text{ (Euclid, VI. A.)}$$
$$:: PM' : PM,$$
$$:: X'N : XN,$$
$$\therefore S'G + SG : S'G - SG :: X'N + XN : X'N - XN;$$
$$\text{or } 2CG : SS' :: 2CN : XX'.$$

Alternately, $2CG : 2CN :: SS' : XX$;

$$\text{or } CG : CN :: CS : CX,$$
$$:: CS^2 : CA^2 \text{ (Prop. II. Cor.)}$$
$$\therefore CG - CN : CN :: CS^2 - CA^2 : CA^2;$$
$$\text{or } NG : CN :: BC^2 : AC^2.$$

## Prop. X.

If $PN$ be the ordinate of any point $P$ on the hyperbola, then

$$PN^2 : AN . A'N :: BC^2 : AC^2.$$

For $NG : NC :: BC^2 : AC^2$.

And rectangles of the same altitude are to one another as their bases, (*Euclid*, VI. 1.)

$$\therefore TN . NG : TN . NC :: BC^2 : AC^2;$$
$$\text{or } PN^2 : TN . NC :: BC^2 : AC^2.$$

But $TN . CN = CN^2 - CT . CN$, (*Euclid*, II. 2.)

$$= CN^2 - CA^2, \text{ (Prop. VIII.)}$$
$$= AN . A'N, \text{ (Euclid, II. 6)}$$
$$\therefore PN^2 : AN . A'N :: BC^2 : AC^2.$$

## Prop. XI.

If the normal at any point $P$ of an hyperbola meet the transverse axis in $G$; then

$$SG : SP :: CS : CA.$$

Produce $S'P$ to $W$; then

since $PG$ bisects the angle $SPW$, (*Prop.* IX.)

$$SG : S'G :: SP : S'P,$$

$$\therefore SG : S'G - SG :: SP : S'P - SP,$$

but $S'P - SP = AA'$, (*Prop.* III.)

and $S'G - SG = SS'$,

$$\therefore SG : SS' :: SP : AA',$$

or $SG : SP :: SS' : AA'$,

or $SG : SP :: CS : CA$.

COR. Hence also,

$$S'G : S'P :: CS : CA.$$

fruit

## PROP. XII.

53. If from the foci $S$ and $S'$ of an hyperbola $SY$ and $S'Y'$ are drawn at right angles to the tangent at $P$, then $Y$ and $Y'$ are on the circumference of the circle described on $AA'$ as diameter, and

$$SY \,.\, S'Y' = BC^2.$$

Join $SP$, $S'P$, and produce $SY$ to meet $S'P$ in $W$; join $CY$; then

since the angle $SPY =$ the angle $WPY$, (*Prop.* VI.)

and the angle $SYP =$ the angle $WYP$,

and the side $PY$ is common to the triangles $SPY$, $WPY$,

$\therefore$ the triangle $SPY = WPY$ in all respects,

$$\therefore SP = PW, \text{ and } SY = WY,$$

$$\therefore S'P - SP = S'W,$$

but $S'P - SP = AA'$, (*Prop.* III.)

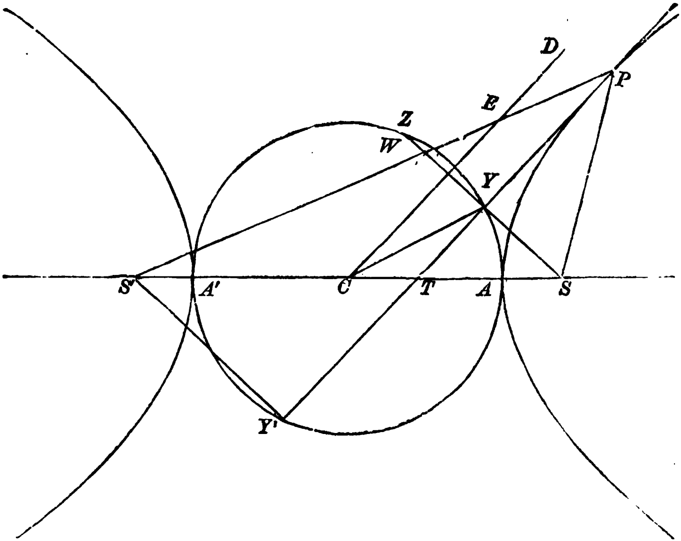
$$\therefore S'W = AA'.$$

Again, $\therefore SC = CS'$, and $SY = WY$,

$$\therefore SC : CS' :: SY : YW,$$

$\therefore CY$ is parallel to $S'W$,

$$\therefore CY : SW :: CS :: SS',$$



$$\therefore\ CY = \tfrac{1}{2}S'W = CA;$$
$$\text{so } CY = CA.$$

$\therefore$ $Y$ and $Y'$ are points on the circumference of the circle described upon $AA'$ as diameter.

Next, let $SY$ be produced to meet this circle in $Z$, and join $ZY'$; then

since the angle $ZYY'$ is a right angle

$\therefore$ $ZY'$ passes through the centre $C$,

$\therefore$ the angle $SCZ$ = the angle $S'CY'$,

$$\begin{aligned} \therefore\ SZ &= S'Y', \\ \therefore\ SY\,.\,S'Y' &= SY\,.\,SZ, \\ &= SA\,.\,SA', \text{ (\textit{Euclid}, III. 36 \textit{Cor.})} \\ &= CS^2 - CA^2, \text{ (\textit{Euclid}, II. 6.)} \\ &= BC^2. \end{aligned}$$

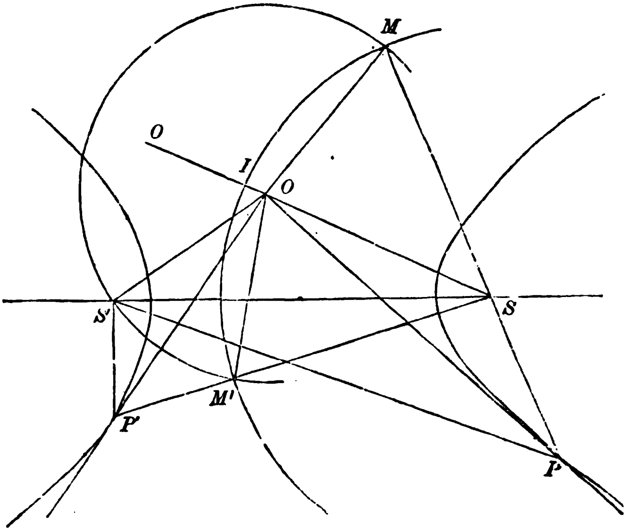
Cor. If $CD$ be drawn parallel to the tangent at $P$ meeting $S'P$ in $E$; then

since $CEPY$ is a parallelogram,

$$\therefore\ PE = CY = AC.$$

### Prop. XIII.

54. To draw a pair of tangents to an hyperbola from an external point $O$.



Of the foci $S$ and $S'$, let $S'$ be that which is nearer to $O$.

With centre $S$ and radius equal to $AA'$ describe a circle.

Join $OS$, $OS'$; and let $SO$ or $S'O$ produced meet the circle in the point $I$.

Now if $O$ be a point inside the circle $MIM'$ it is evident that $OS'$ is greater than $OI$; and if $O$ be outside the circle,

$$\text{since } OS - OS' < AA' \text{ or } SI, \text{ (\textit{Prop.} IV.)}$$

$$\therefore OS - OS' < OS - OI,$$

$$\therefore OS' > OI.$$

With centre $O$ and radius $OS'$ describe another circle cutting the former in the points $M$ and $M'$, which it will always do since $OS'$ is greater than $OI$.

Join $SM$, $SM'$, and produce them to meet the hyperbola in the points $P$ and $P'$.

Join $OP$, $OP'$; these will be the tangents required.

Join $S'P$, $S'P'$; then

$$\text{since } S'P - SP = AA' = SM,$$
$$\therefore\ S'P = PM.$$
$$\text{And } \therefore\ S'P,\ PO = MP,\ PO, \text{ each to each,}$$
$$\text{and } OS'' = OM,$$
$$\therefore \text{ the angle } OPS' = \text{the angle } OPM,$$
$$\therefore\ OP \text{ is the tangent at } P. \quad (\textit{Prop. } \text{VI.})$$
$$\text{So } OP \text{ is the tangent at } P'.$$

The points of contact $P$ and $P'$ will be upon the same or opposite branches of the hyperbola according as $SM$ and $SM'$ require to be produced in the same or in opposite directions with respect to $S$, in order to intersect the hyperbola.

## PROP. XIV.

Omit

If from a point $O$ a pair of tangents, $OP$, $OP'$ be drawn to an hyperbola, then the angles which $OP$ and $OP'$ subtend at either focus will be equal or supplementary according as the points of contact are in the same or opposite branches of the hyperbola.

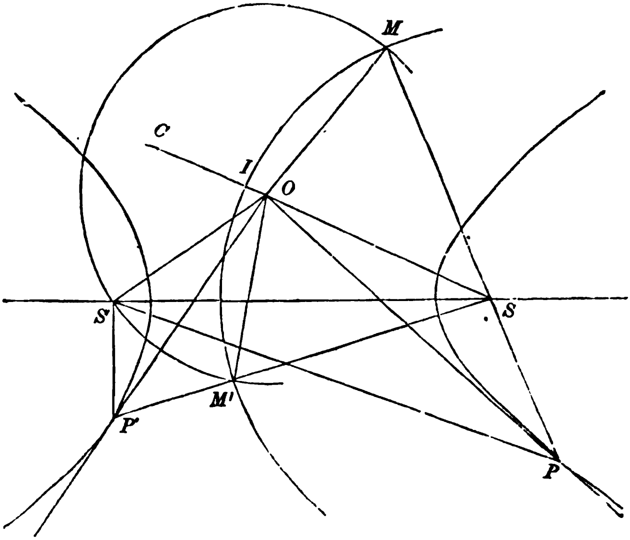
Let the points $P$ and $P'$ be on opposite branches of the hyperbola.

Join $PS$, $S'P$; $SP'$, $S'P'$.

Produce $PS$ to $M$, making $PM$ equal to $PS'$. Also from $P'S$ cut off a part $P'M'$ equal to $P'S'$.

Join $OM$, $OM'$; $OS$, $OS'$.

$$\text{Then since } OP,\ PS' = OP,\ PM, \text{ each to each,}$$
$$\text{and the angle } OPS' = \text{the angle } OPM, \ (\textit{Prop. } \text{VI.})$$
$$\therefore\ OS' = OM,$$
$$\text{and the angle } OS'P = \text{the angle } OMP.$$



So $OS' = OM'$,

and the angle $OS'P'$ = the angle $OM'P'$,

$\therefore OM = OM'$.

Again, $\because SM = S'P - SP = AA'$,

and $SM' = SP' - S'P' = AA'$,

$\therefore SM = SM'$.

And $\because OS, SM = OS, SM'$, each to each,

and $OM = OM'$,

$\therefore$ the angle $OSM$ = the angle $OSM'$,

and the angle $OMS$ = the angle $OM'S$.

But $OSM$ is the supplement of $OSP$,

and $OM'S$ is the supplement of $OM'P'$,

$\therefore OSM'$ is the supplement of $OSP$,

and $OMP$ the supplement of $OM'P'$.

But $OMP = OS'P$,

and $OM'P' = OS'P'$,

$\therefore OS'P$ is the supplement of $OS'P$.

Hence the angles which $OP$ and $OP'$ subtend either at $S$ or $S'$ are supplementary.

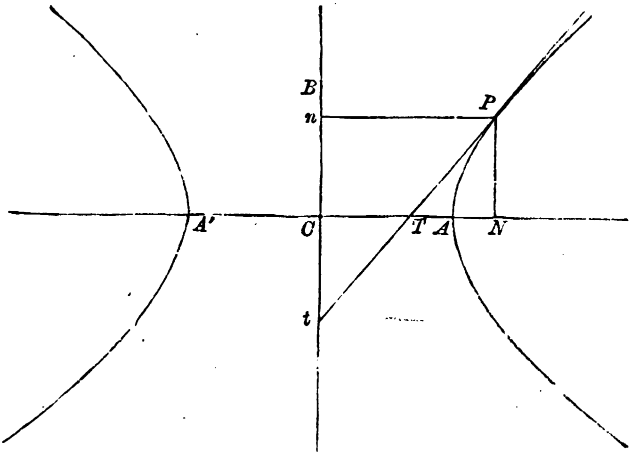
In a similar manner if $P$ and $P'$ are on the same branch of the hyperbola, the angles subtended either at $S$ or $S'$ may be shown to be equal.

## PROP. XV.

Omit

55. If the tangent at any point $P$ of an hyperbola meet the conjugate axis in the point $t$, and $Pn$ be drawn at right angles to $CB$; then

$$Cn \,.\, Ct = BC^2.$$



Draw $PN$ at right angles to $CA$; then

$$Ct : CT :: PN : NT,$$
$$\therefore Ct : PN :: CT : NT,$$
$$\therefore Ct \,.\, Cn : PN^2 :: CT \,.\, CN : CN \,.\, NT;$$
$$\text{or } Ct \,.\, Cn : CT \,.\, CN :: PN^2 : CN \,.\, NT,$$
$$:: BC^2 : AC^2. \quad (\textit{Prop. X.})$$
$$\text{But } CT \,.\, CN = AC^2,$$
$$\therefore Ct \,.\, Cn = BC^2.$$

56. The proofs that we have given up to this point of the properties of the hyperbola are closely analogous to the corresponding propositions in the ellipse. The remaining properties of the hyperbola are more conveniently investigated by means of its relation to certain lines, which we shall presently define, called *Asymptotes*, in the same manner as many of the properties of the ellipse were deduced from those of the auxiliary circle.

Def. The hyperbola described (*see fig. Prop.* XIV.) with $C$ as centre, and $BB'$ as transverse axis, and $AA'$ as conjugate axis, is called the *Conjugate Hyperbola.* Its foci, which will be on the line $BCB'$, will evidently be at the same distance from $C$ as those of the original hyperbola, since

$$CS^2 = CA^2 + CB^2.$$

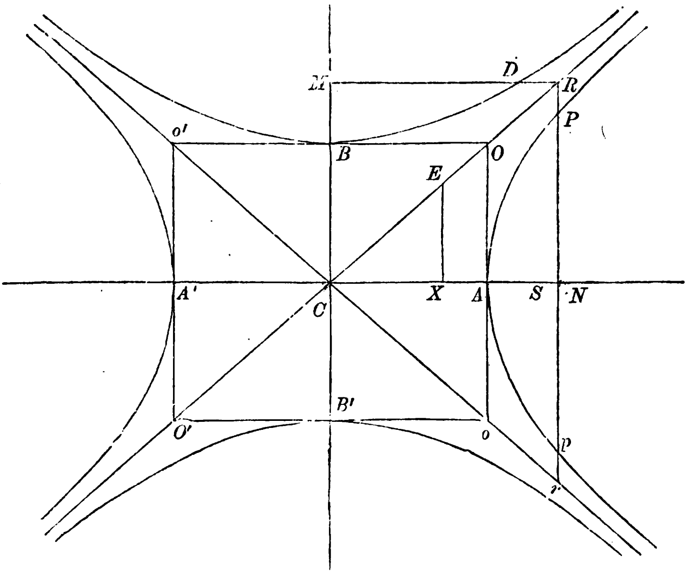
## Prop. XVI.

If through any point $R$ or either of the diagonals of the rectangle formed by drawing tangents to the hyperbola and its conjugate at the vertices, $A$, $A'$, $B$, $B'$, two ordinates $RPN$, $RDM$, be drawn at right angles to $AA'$ and $BB'$, and meeting either the hyperbola or its conjugate in the points $P$ and $D$; then

$$RN^2 \backsim PN^2 = BC^2,$$
$$\text{and } RM^2 \backsim DM^2 = AC^2.$$

Let $R$ be a point on the diagonal $O'CO$; then

$$\begin{aligned} RN^2 : CN^2 &:: AO^2 : AC^2, \\ &:: BC^2 : AC^2, \\ \text{and } PN^2 : CN^2 - CA^2 &:: BC^2 : AC^2; \; (\textit{Prop. } \text{X.}) \\ \therefore RN^2 - PN^2 : CA^2 &:: BC^2 : AC^2; \\ \therefore RN^2 - PN^2 &= BC^2. \\ \text{Again, } RM^2 : CM^2 &:: AC^2 : BC^2, \\ \text{and } DM^2 : CM^2 - CB^2 &:: AC^2 : BC^2; \; (\textit{Prop. } \text{X.}) \\ \therefore RM^2 - DM^2 : BC^2 &:: AC^2 : BC^2; \\ \therefore RM^2 - DM^2 &= AC^2. \end{aligned}$$



In exactly the same manner, if $NR$ had been produced to meet the conjugate hyperbola in $P$, and $MR$ had been produced to meet the original hyperbola in $D$, we should have had,

$$PN^2 - RN^2 = BC^2,$$

$$\text{and } DM^2 - RM^2 = AC^2.$$

COR. If $RP$ be produced to meet the hyperbola in $p$, and the other asymptote in $r$; then

$$RN^2 - PN^2 = RP \,.\, Pr; \text{ (\textit{Euclid}, II. 5.)}$$

$$\therefore RP \,.\, Pr = BC^2.$$

Hence as $RPN$ is further removed from $A$, and the line $Pr$ consequently increases, since the rectangle contained by $RP$ and $Pr$ remains constant, $RP$ must diminish, and by taking $R$ sufficiently far from $C$, $RP$ may be made less than any assignable magnitude. The line $CR$, therefore, continually approaches nearer and nearer to the hyperbola, though it never actually reaches it.

On account of this property, $CR$ is called an *Asymptote* to the hyperbola.

So also if $P$ be the point where $NR$ produced meets the conjugate hyperbola, we shall have

$$RP \,.\, Pr = BC^2;$$

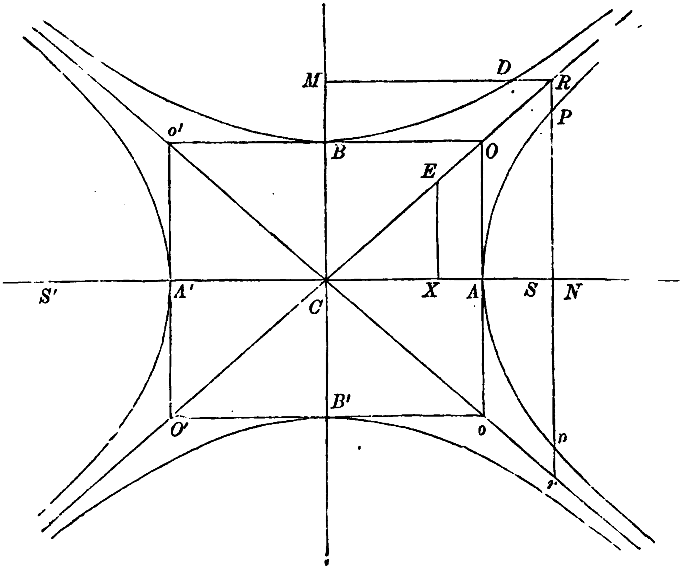
and therefore $CR$ is also the asymptote to the conjugate hyperbola.

In the same manner it may be shown that the other diameter $oCo'$ of the rectangle $OO'$ is an asymptote to both hyperbolas.

## PROP. XVII.

57. If $E$ be the point where the asymptote meets the directrix; then

$$CE = AC.$$



For by similar triangles,

$$CE : CO :: CX : CA,$$

$$:: CA : CS. \quad (\textit{Prop.}\ \text{II.})$$

$$\text{But } CO^2 = CA^2 + CB^2 = CS^2;$$

$$\therefore CO = CS;$$

$$\therefore CE = AC.$$

Cor. If $SE$ be joined, since

$$CE^2 = CA^2 = CS . CX,$$

$\therefore$ the angle $CES$ is a right angle. (*Euclid*, VI. 8, *Cor.*)

## Prop. XVIII.

If from any point $R$ in one of the asymptotes to an hyperbola ordinates $RPN$, $RDM$ be drawn to the hyperbola and its conjugate respectively, and $PD$ be joined, $PD$ will be parallel to the other asymptote.

$$\text{For } RN^2 : RM^2 :: BC^2 : AC^2;$$

$$\text{and } RN^2 - PN^2 : RM^2 - DM^2 :: BC^2 : AC^2, \quad (\textit{Prop.}\ \text{XVI.})$$

$$\therefore PN^2 : DM^2 :: BC^2 : AC^4;$$

$$:: RN^2 : RM^2,$$

$$\therefore PN : DM :: RN : RM;$$

$$\therefore PD \text{ is parallel to } MN. \quad (\textit{Euclid},\ \text{VI. 2.})$$

$$\text{Also } CN : CM :: AC : BC,$$

$$\therefore MN \text{ is parallel to } AB;$$

$$\text{and } OA : Ao :: OB : Bo',$$

$$\therefore AB \text{ is parallel to } oo'.$$

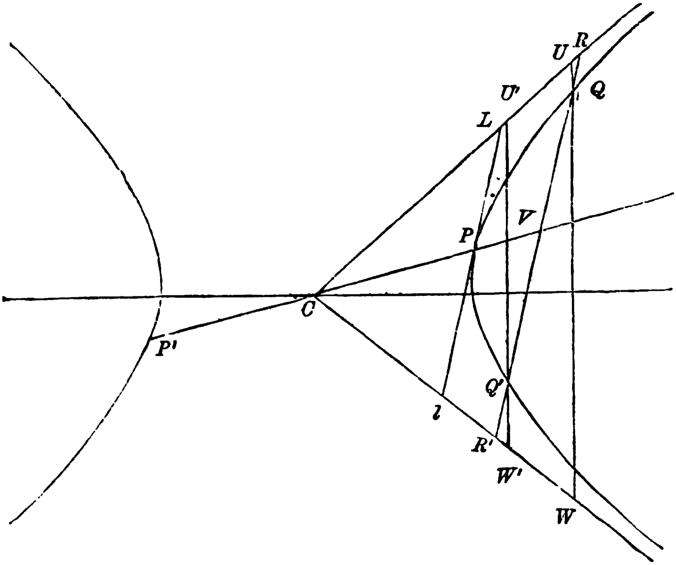
$$\text{Hence } PD \text{ is parallel to } oo'.$$

Cor. So also if $R$ and $D$ be the points where $NR$ and $MR$ produced meet respectively the conjugate and the original hyperbola, $PD$ will be still parallel to $oo'$.

## Prop. XIX.

58. If through any two points $Q$ and $Q'$ of an hyperbola a line $RQQ'R'$ be drawn in any direction meeting the asymptotes in $R$ and $R'$; then will

$$RQ = R'Q'.$$



Through $Q$ and $Q'$ draw the ordinates $UQW$, $U'Q'W'$; meeting the asymptotes in $U$, $W$, $U'$, $W'$; then by similar triangles,

$$QR : QU :: Q'R : Q'U',$$

$$\text{and } QR' : QW :: Q'R' : Q'W';$$

$\therefore$ compounding

$$QR \,.\, QR' : QU \,.\, QW :: Q'R \,.\, Q'R' : Q'U' \,.\, Q'W'.$$

$$\text{But } QU \,.\, QW = BC^2 = Q'U' \,.\, Q'W, \quad (\textit{Prop. } \text{XVI. } \textit{Cor.})$$

$$\therefore QR \,.\, QR' = Q'R \,.\, Q'R';$$

$$\text{but } QR \,.\, QR' = QR \,.\, QQ' + QR \,.\, Q'R',$$
$$\text{and } Q'R \,.\, Q'R' = Q'R' \,.\, QQ' + QR \,.\, Q'R';$$
$$\therefore\ QR \,.\, QQ' = Q'R' \,.\, QQ'.$$
$$\therefore\ QR = Q'R'.$$

Cor. 1. If $RQQ'R'$ move parallel to itself until the points $Q$ and $Q'$ coincide, the line $RQR'$ will ultimately assume the position $LPl$, and will become a tangent to the hyperbola at $P$.

Hence, since $RQ$ is always equal to $R'Q'$,

$$LP = Pl,$$

or the tangent $LPl$ is bisected at the point of contact $P$.

Cor. 2. If $CP$ be produced to meet $RR'$ in $V$, then since

$$RV : VR' :: LP : Pl,$$
$$\therefore RV = VR';$$
$$\text{and } RQ = Q'R',$$
$$\therefore QV = Q'V.$$

Hence, if a series of parallel chords be drawn in an hyperbola, their middle points will all be in the line drawn through the centre and the point where the tangent parallel to the chords meets the hyperbola.

Def. A line $PCP'$ drawn through the centre, and meeting the hyperbola in $P$ and $P'$, is called a *Diameter*.

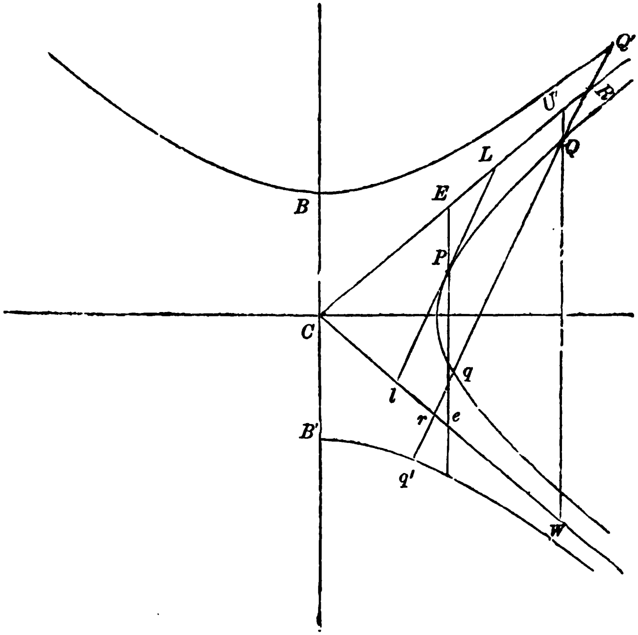
A diameter consequently bisects all chords drawn parallel to the tangents at its extremities.

### Prop. XX.

59. If through any point $Q$ of an hyperbola a line $RQr$ be drawn in any direction meeting the asymptotes in $R$ and $r$, and $LPl$ be the tangent drawn parallel to $RQr$; then

$$RQ \,.\, Qr = PL^2.$$

Through $P$ and $Q$ draw the ordinates $EPe$, $UQW$, meeting the asymptote in $E$, $e$, $U$, $W$; then by similar triangles,



$$QR : QU :: PL : PE,$$

$$Qr : QW :: Pl : Pe;$$

$$\therefore QR \,.\, Qr : QU \,.\, QW :: PL \,.\, Pl : PE \,.\, Pe;$$

but $QU \,.\, QW = BC^2 = PE \,.\, Pe$, (*Prop.* XVI. *Cor.*)

$$\therefore QR \,.\, Qr = PL \,.\, Pl,$$

$$= PL^2. \quad (\textit{Prop.}\ \text{XIX.}\ \textit{Cor.}\ 1.)$$

Cor. If $Qq$ be produced to meet the conjugate hyperbola in $Q'$, $q'$, we may show that

$$Q'R \,.\, Q'r = PL^2,$$

and also, as in Proposition XIX., that

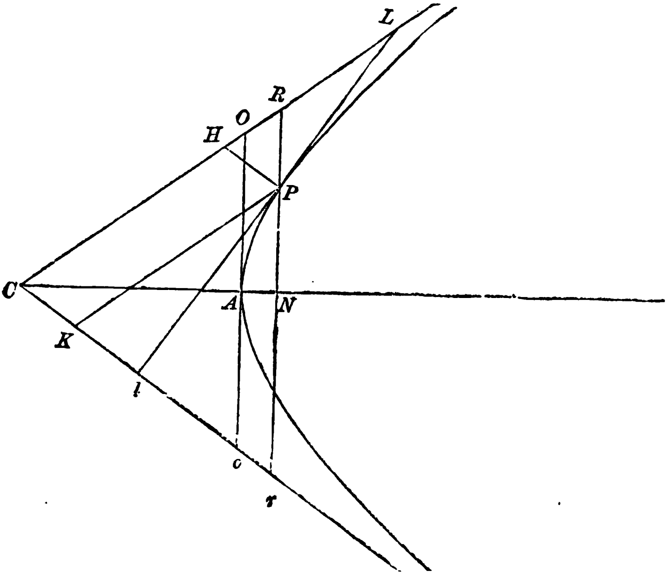
$$Q'R = q'r,$$

$$\therefore QQ' = qq'.$$

Hence if a line be drawn in any direction meeting both the hyperbolas, the portions intercepted between the hyperbola and its conjugate will be equal.

## PROP. XXI.

60. If from any point $P$ of an hyperbola, $PH$ and $PK$ be drawn parallel to the asymptotes, meeting them in $H$ and $K$ respectively; then $4 . PH : PK = CS^2$.



Draw the ordinate $RPNr$ meeting the asymptotes in $R$ and $r$; then by similar triangles,

$$PH : PR :: Co : Oo,$$

$$\text{and } PK : Pr :: CO : Oo,$$

$$\therefore PH . PK : PR . Pr :: CO^2 : Oo^2,$$

$$:: CS^2 : 4BC^2.$$

$$\text{But } PR . Pr = BC^2,$$

$$\therefore 4 . PH . PK = CS^2.$$

## PROP. XXII.

If the tangent at any point $P$ of an hyperbola meet the asymptotes in $L$ and $l$; then the area of the triangle $LCl$ is equal to the rectangle contained by $AC$ and $BC$.

Draw $PH$ and $PK$ parallel to the asymptotes meeting them in $H$ and $K$; then

$$\text{since } CL : CH :: Ll : Pl,$$

$$\text{and } Ll = 2Pl, \quad (\textit{Prop.}\ \text{XIX. } \textit{Cor.}\ 1.)$$

$$\therefore CL = 2CH = 2PK;$$

$$\text{so } Cl = 2CK = 2PH,$$

$$\therefore CL \,.\, Cl = 4PH \,.\, PK = CS^2, \quad (\textit{Prop.}\ \text{XXI.})$$

$$= CO \,.\, Co,$$

$$\therefore CL : CO :: Co : Cl,$$

$\therefore$ the triangles $LCl$, $OCo$ have the angle at $C$ common and the sides about those angles reciprocally proportional·

$$\therefore \text{the triangle } LCl = \text{the triangle } OCo,$$

$$= AC \,.\, AO.$$

$$= AC \,.\, BC.$$

## PROP. XXIII.

61. If from any point $R$ in the asymptote of an hyperbola two ordinates $RPN$ and $RDM$ be drawn to the hyperbola and its conjugate respectively, then the tangents at $P$ and $D$ will be parallel respectively to $CD$ and $CP$.

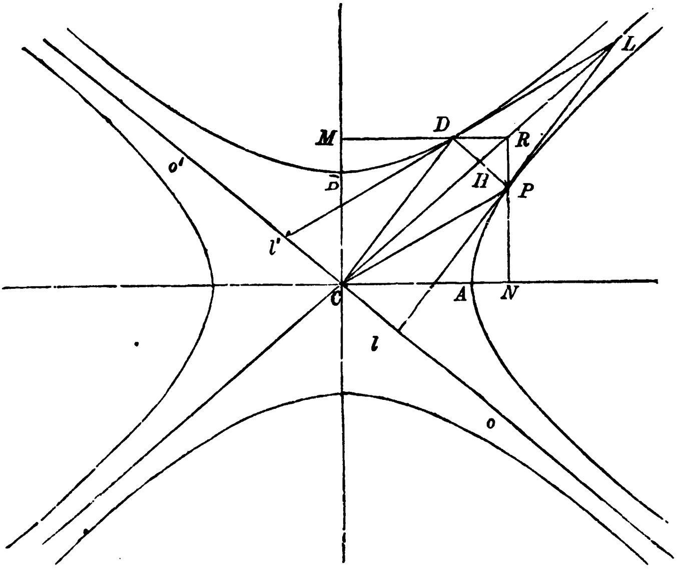
Join $PD$, meeting $CR$ in $H$; then

$$\text{since } PD \text{ is parallel to } oo', \quad (\textit{Prop.}\ \text{XVIII.})$$

the tangents at $P$ and $D$ will each meet $CR$ produced in the same point $L$. (*Prop.* XXII.)

Produce $LP$ and $LD$ to meet the other asymptotes in $l$ and $l'$; then

$$\text{since } CL \,.\, Cl = CS^2 = CL \,.\, Cl', \quad (\textit{Prop.}\ \text{XXII.})$$



$$\therefore\ Cl = Cl',$$

$$\therefore\ lC : Cl' :: lP : PL,$$

$\therefore$ $CP$ is parallel to the tangent at $D$.

Also $l'D : DL :: l'C : Cl$,

$\therefore$ $CD$ is parallel to the tangent at $P$.

The lines $CP$ and $CD$ are called *Conjugate Diameters*, since each of these lines is parallel to the tangent at the extremity of the other.

## Prop. XXIV.

If $CP$ and $CD$ be semi-conjugate diameters in the hyperbola; then

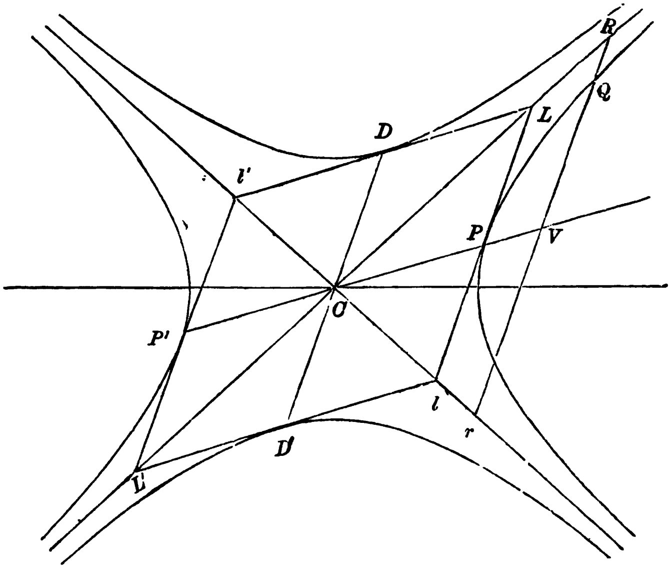
$$CP^2 \sim CD^2 = CA^2 \sim CB^2.$$

Draw the ordinates $NPR$, $MDR$ meeting the asymptote in the point $R$ (*Prop.* XXIII.); then

$$\begin{aligned} CR^2 - CP^2 &= NR^2 - NP^2, \\ &= BC^2, \quad (\textit{Prop.}\ \text{XVI.}) \\ \therefore\ CR^2 &= CP^2 + BC^2; \\ \text{so } CR^2 &= CD^2 + AC^2, \\ \therefore\ CP^2 + BC^2 &= CD^2 + AC^2; \\ \text{or } CP^2 \sim CD^2 &= AC^2 \sim BC^2. \end{aligned}$$

### PROP. XXV.

62. The area of any parallelogram formed by drawing tangents to the hyperbola and its conjugate at the extremities $P, P', D, D'$ of a pair of conjugate diameters is equal to the rectangle contained by the axes.



Let $Ll$, $L'l'$ be the parallelogram formed by drawing tangents at the extremities $P$, $P'$, $D$, $D'$, of any pair of conjugate diameters. The points $L$, $L'$, $l$, $l'$, will (*Prop.* XXIII.) be on the asymptotes.

Now the parallelogram $LL' = 4$ parallelogram $CL$,

$$\begin{aligned} &= 4 \text{ triangle } LCl, \\ &= 4\,AC\,.\,BC, \quad (\textit{Prop.}\ \text{XXII.}) \\ &= AA'\,.\,BB'. \end{aligned}$$

COR. If $PF$ be drawn perpendicular to $CD$, then

$$PF\,.\,CD = AC\,.\,BC.$$

Also, if the normal $PG$ meet the transverse axis in $G$, as in the ellipse

$$PF\,.\,PG = BC^2.$$

63. DEF. The line $QV$ drawn from any point $Q$ of the hyperbola parallel to the tangent at any point $P$, and meeting $CP$ produced in $V$, is called an *Ordinate* to the diameter $CP$.

## PROP. XXVI.

If $QV$ be an ordinate to the diameter $P'CP$, and $CD$ be conjugate to $CP$; then

$$QV^2 : PV\,.\,P'V :: CD^2 : CP^2.$$

Produce $VQ$ to meet the asymptotes in $R$ and $r$; and let the tangent at $P$ meet the asymptotes in $L$ and $l$; then

$$RV^2 : PL^2 :: CV^2 : CP^2,$$
$$\therefore\ RV^2 - PL^2 : PL^2 :: CV^2 - CP^2 : CP^2.$$

But $RQ\,.\,Qr = PL^2$, (*Prop.* XX.)

$$\therefore\ RV^2 - QV^2 = PL^2,$$
$$\text{or } RV^2 - PL^2 = QV^2.$$

And $CV^2 - CP^2 = PV\,.\,P'V$, (*Euclid*, II. 6.)

$$\therefore\ QV^2 : PL^2 :: PV\,.\,P'V : CP^2.$$

Alternately, $QV^2 : PV\,.\,P'V :: PL^2 : CP^2$.

But since $PD$ is a parallelogram, (*Prop.* XXIII.)

$$\therefore\ PL = CD.$$

Hence $QV^2 : PV\,.\,P'V :: CD^2 : CP^2$.

COR. If $VQ$ be produced to meet the conjugate hyperbola in $Q'$, then

since $Q'R\,.\,Q'r = PL^2$, (*Prop.* XX. *Cor.*)

$$\therefore\ Q'V^2 - RV^2 = PL^2.$$

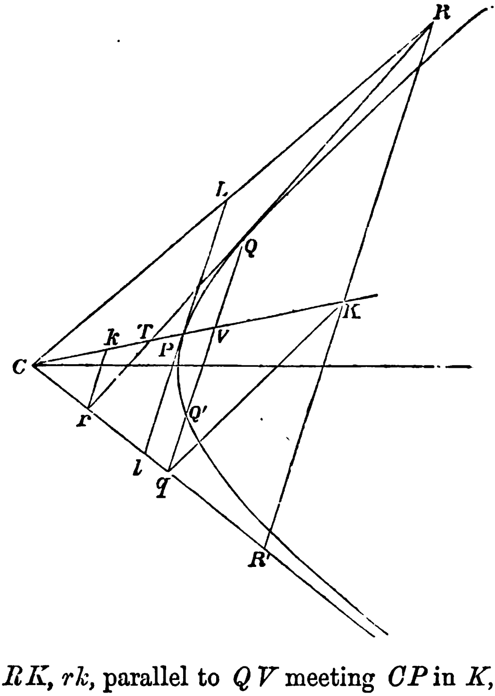
Hence $Q'V^2 : CV^2 + CP^2 :: CD^2 : CP^2$.

## PROP. XXVII.

64. If $QV$ be an ordinate to the diameter $PV$, and the tangent at $Q$ meet $CP$ in the point $T$; then

$$CV \cdot CT = CP^2.$$

Draw the tangent $LPl$ meeting the asymptotes in the points $L$, $l$; also let the tangent at $Q$ meet the asymptotes in $R$, $r$.



Draw $RK$, $rk$, parallel to $QV$ meeting $CP$ in $K$, $k$.

Now since the triangles $RCr$, $LCl$ are equal, (*Prop.* XXII.) and have the angle at $C$ common,

$\therefore\ CR : CL :: Cl : Cr.$ (*Euclid*, VI. 15.)

But $CR : CL :: CK : CP$,

$$\text{and } Cl : Cr :: CP : Ck,$$

$$\therefore CK : CP :: CP : Ck,$$

$$\therefore CK \,.\, Ck = CP^2.$$

Again, produce $RK$ and $QV$ to meet the asymptote $Cl$ in $R'$ and $q$; then

Since $Rr$ is bisected in $Q$, (*Prop.* XIX. *Cor.* 1.)

$$\therefore R'r \text{ is bisected in } q,$$

$$\text{and } RK = R'K, \text{ (Prop. XIX. Cor. 2.)}$$

$$\therefore Kq \text{ is parallel to } Rr,$$

$$\therefore CT : CK :: Cr : Cq,$$

$$:: Ck : CV,$$

$$\therefore CV \,.\, CT = CK \,.\, Ck$$

$$= CP^2.$$

Cor. 1. Conversely, if $QV$ be an ordinate to $PV$,

and $CV \,.\, CT = CP^2$,

then $QT$ is the tangent at $Q$.

Cor. 2. Hence also, if $RR'$ meet the curve in $U$ and $U'$,

and $kU$, $kU'$ be drawn,

since $CK \,.\, Ck = CP^2$,

$\therefore$ $kU$ and $kU'$ are tangents to the hyperbola at $U$ and $U'$.

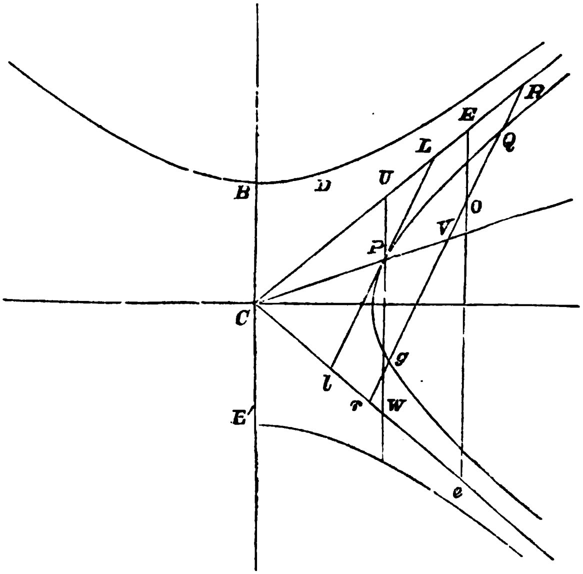
## Prop. XXVIII.

65. If two chords of a hyperbola intersect one another, the rectangles contained by their segments are proportional to the squares of the diameters parallel to them.

Let $QOq$ be any chord drawn through the point $O$, and let $CD$ be drawn parallel to it, meeting the conjugate hyperbola in $D$.

Produce $Qq$ to meet the asymptotes in $R$ and $r$; and draw the diameter $CPV$, bisecting both $Qq$ and $Rr$ in $V$. (*Prop.* XIX. *Cor.* 2.)

Also draw the tangent $LPl$ parallel to $Qq$, meeting the asymptotes in $L$ and $l$.



Now since $Qq$ is divided equally in $V$ and unequally in $O$,

$$\therefore\ QO\,.\,Oq = QV^2 - OV^2;\ (\textit{Euclid}, \text{II. } 5.)$$

so also $RO\,.\,Or = RV^2 - OV^2$; (*Euclid*, II. 5.)

$$\therefore\ RO\,.\,Or - QO\,.\,Oq = RV^2 - QV^2,$$

$$= RQ\,.\,Qr\ (\textit{Euclid}, \text{II. } 5.)$$

$$= PL^2,\ (\textit{Prop.}\ \text{XX.})$$

$$\therefore\ QO\,.\,Oq = RO\,.\,Or - PL^2.$$

Again, through $O$ and $P$ draw $EOe$, $UPW$, at right angles to the axis meeting the asymptotes in $E$, $e$, $U$, $W$; then

$$RO : OE :: PL : PU,$$

and $rO : Oe :: Pl : PW$,

$$\therefore\ RO\,.\,rO : OE\,.\,Oe :: PL^2 : PU\,.\,PW;$$

$$\text{but } PU \,.\, PW = BC^2, \ (\textit{Prop.}\ \text{XVI.})$$
$$\text{and } PL^2 = CD^2, \ (\textit{Prop.}\ \text{XXIII.})$$
$$\therefore RO \,.\, rO : OE \,.\, Oe :: CD^2 : BC^2,$$
$$\text{or } RO \,.\, rO : CD^2 :: OE \,.\, Oe : BC^2.$$
$$\therefore RO \,.\, rO - PL^2 : CD^2 :: OE \,.\, Oe - BC^2 : BC^2,$$
$$\text{or } QO \,.\, Oq : CD^2 :: OE \,.\, Oe - BC^2 : BC^2.$$

In the same manner if through $O$ another chord $Q'\ Oq'$ be drawn, and $CD'$ be drawn parallel to it, meeting the conjugate hyperbola in $D'$, we shall have

$$Q'O \,.\, Oq' : CD'^2 :: OE \,.\, Oe - BC^2 : BC^2.$$

Hence $QO \,.\, Oq : Q'O \,.\, Oq' :: CD^2 : CD'^2$.

COR. The same result may be shown to be true when the point $O$ is outside the hyperbola. Moreover, it is not necessary that the chords should be drawn meeting *one* branch only of the hyperbola or the *same* branch. The proportion still holds good when one or both of the chords meet both branches of the hyperbola, or when the chords are drawn in different branches.

66. DEF. If with a point $O$ on the normal at $P$ as centre, and $OP$ as radius, a circle be described touching the hyperbola at $P$, and cutting it in $Q$; then when the point $Q$ is made to approach indefinitely near to $P$, the circle is called the *Circle of Curvature* at the point $P$.

## PROP. XXIX.

If $PH$ be the chord of the circle of curvature at the point $P$ of a hyperbola, which passes through the centre; then

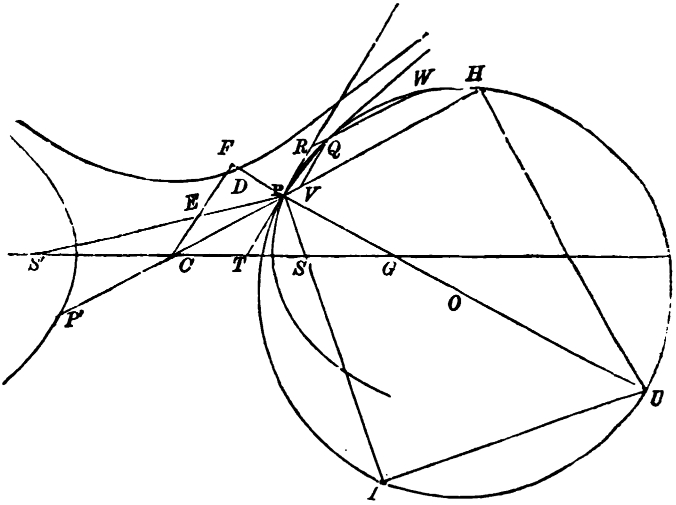
$$PH \,.\, CP = 2CD^2.$$

Let $PT$ be the tangent, and $PG$ the normal at the point $P$.

With centre $O$, and radius $OP$, describe a circle cutting the hyperbola in the point $Q$.

Draw $RQW$ parallel to $CP$, meeting the circle in $W$, and $TP$ produced in $R$.

Also, draw $QV$ parallel to $PR$, meeting the diameter $PP'$ in $V$; then, since $RP$ touches the circle at $P$,



$$\therefore\ RQ \,.\, RW = PR^2, \text{ (\textit{Euclid}, III. 36.)}$$
$$\text{or } PV \,.\, RW = QV^2.$$
$$\text{But } QV^2 : PV \,.\, P'V :: CD^2 : CP^2, \text{ (\textit{Prop.} XXVI.)}$$
$$\therefore\ PV \,.\, RW : PV \,.\, P'V :: CD^2 : CP^2,$$
$$\text{or } RW : P'V :: CD^2 : CP^2.$$

Now, when the circle becomes the circle of curvature at $P$, the points $R$ and $Q$ move up to, and coincide with $P$, and the lines $RW$ and $PH$ become equal, while

$$P'V \text{ becomes equal to } PP', \text{ or } 2\,CP.$$
$$\text{Hence, } PH : 2\,CP :: CD^2 : CP^2,$$
$$\therefore\ PH \,.\, CP : 2\,CP^2 :: 2\,CD^2 : 2\,CP^2,$$
$$\therefore\ PH \,.\, CP = 2\,CD^2.$$

## PROP. XXX.

If $PU$ be the diameter of the circle of curvature at the point $P$ of the hyperbola, and $PF$ be drawn at right angles to $CD$; then

$$PU \,.\, PF = 2\,CD^2.$$

Since the triangle $PHU$ is similar to the triangle $PFC$,

$$\therefore PU : PH :: CP : PF,$$
$$\therefore PU \,.\, PF = PH : CP,$$
$$= 2CD^2. \quad (\textit{Prop.}\ \text{XXIX.})$$

## Prop. XXXI.

If $PI$ be the chord of the circle of curvature through the focus of the hyperbola; then

$$PI \,.\, AC = 2CD^2.$$

Let $S'P$ meet $CD$ in $E$; then, since the triangles $PIU$ and $PEF$ are similar,

$$\therefore PI : PU :: PF : PE.$$
$$\text{But } PE = AC, \quad (\textit{Prop.}\ \text{XII. } \textit{Cor.})$$
$$\therefore PI : PU :: PF : AC,$$
$$\therefore PI \,.\, AC = PU \,.\, PF,$$
$$= 2CD^2. \quad (\textit{Prop.}\ \text{XXX.})$$

The point where the circle of curvature intersects the hyperbola may be determined as in the case of the ellipse.

## Prop. XXXII.

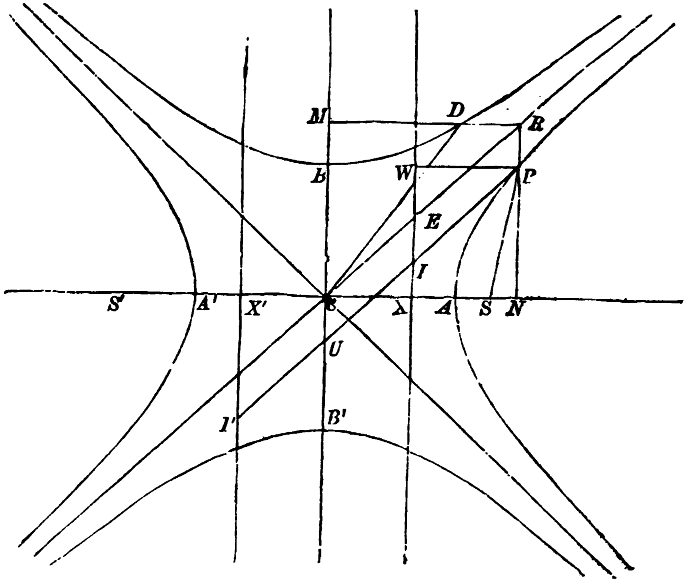
67. If $P$ be any point on the hyperbola, and $CD$ be conjugate to $CP$; then

$$SP \,.\, S'P = CD^2.$$

Draw $PII'$ parallel to the asymptote $CE$ meeting the directrices in $I$ and $I'$, and $CB'$ in $U$.

Let the ordinates, $NP$, $MD$ meet the asymptote in $R$, and draw $PW$ perpendicular to the directrix; then by similar triangles,

$$PI : PW :: CE : CX,$$
$$:: CA : CX. \quad (\textit{Prop.}\ \text{XVII.})$$



But $SP : PW :: SA : AX$,

$:: CA : CX.$

$\therefore SP = PI$;

so $S'P = PI'$,

$\therefore SP \,.\, S'P = PI \,.\, PI'$,

$= UP^2 - UI^2$,

$= CR^2 - CE^2$,

$= CR^2 - CA^2$. (*Prop.* XVII.)

But $CR^2 - CD^2 = RM^2 - DM^2$,

$= CA^2$, (*Prop.* XVI.)

$\therefore CR^2 - CA^2 = CD^2$,

Hence $SP \,.\, S'P = CD^2$.

## PROBLEMS ON THE HYPERBOLA.

1. The locus of the centre of a circle touching two given circles is an hyperbola or ellipse.

2. If on the portion of any tangent intercepted between the tangents at the vertices a circle be described, it will pass through the foci.

3. In an hyperbola the tangents at the vertices will meet the asymptotes in the circumference of the circle described on $SS'$ as diameter.

4. If from a point $P$ in an hyperbola $PII'$ be drawn parallel to the transverse axis meeting the asymptotes in $I$ and $I'$, then $PI \,.\, PI' = AC^2$.

5. If a circle be inscribed in the triangle $SPS'$, the locus of its centre is the tangent at the vertex.

6. If $PN$ be the ordinate of the point $P$, and $NQ$ a tangent to the circle described on the transverse axis as diameter, and $PM$ be drawn parallel to $QC$ meeting the axis in $M$, then $MN = BC$.

7. If $PN$ be the ordinate of a point $P$, and $NQ$ be drawn parallel to $AP$ to meet $CP$ in $Q$, then $AQ$ is parallel to the tangent at $P$.

8. If an hyperbola and an ellipse have the same foci, they cut one another at right angles.

9. If the tangent at $P$ intersect the tangents at the vertices in $R$, $r$, and the tangent at $P'$ intersect them in $R'$, $r'$, then $AR \,.\, Ar = AR' \,.\, Ar'$.

10. If any two tangents be drawn to an hyperbola, the lines joining the points where they intersect the asymptotes will be parallel.

11. The perpendicular drawn from the focus to the asymptotes of an hyperbola is equal to the semi-conjugate axis.

12. If the asymptotes meet the tangent at the vertex in $O$, and the directrix in $E$; then $AE$ is parallel to $SO$.

13. In a rectangular hyperbola conjugate diameters are equal to one another.

14. In a rectangular hyperbola the normal $PG$ is equal to $CP$.

15. The lines drawn from any point in a rectangular hyperbola to the extremities of a diameter make equal angles with the asymptotes.

16. Prove that the asymptotes to an hyperbola bisect the lines joining the extremities of conjugate diameters.

17. A line drawn through one of the vertices of an hyperbola and terminated by two lines drawn through the other vertex parallel to the asymptotes will be bisected at the other point where it cuts the hyperbola.

18. $P$ is any point on an hyperbola, and $P'$ a point on the conjugate hyperbola. If $CP$ and $CP'$ be conjugate, prove that

$$S'P' - SP = AC - BC,$$

$S$ and $S'$ being the interior foci.

19. If $CP$ and $CD$ be conjugate, and through $C$ a line be drawn parallel to either focal distance of $P$, the perpendicular from $D$ upon this line is equal to $BC$.

20. Given a pair of conjugate diameters, find the principal axes.

21. If $Q$ be a point on the conjugate axis of a rectangular hyperbola, and $QP$ be drawn parallel to the transverse axis meeting the curve in $P$, then

$$PQ = AQ.$$

22. In a rectangular hyperbola the focal chords drawn parallel to conjugate diameters are equal.

23. If in an equilateral hyperbola $CY$ be drawn at right angles to the tangent at $P$, and $AY$ be joined, the triangles $PCA$, $CAY$ are similar.

24. The radius of the circle which touches an hyperbola and its asymptotes, is equal to that part of the latus rectum produced which is intercepted between the curve and the asymptotes.

25. If $QQ'$ be any chord of an hyperbola, and $CP$ the diameter corresponding to it, and $QH$, $PK$, $Q'H'$ be drawn parallel to one asymptote meeting the other in $H$, $K$ and $H'$, then $CH \,.\, CH' = CK^2$.

26. If the chord $RPP'R'$ intersect the hyperbola in the points $P$, $P'$, and the asymptotes in $R$, $R'$; and $PK$ be drawn parallel to $CR'$, and $P'K'$ to $CR$; then $RK = P'K'$, and $R'K' = PK$.

27. If $AA'$ be any diameter of a circle, and $PNQ$ an ordinate to it, then the locus of the intersections of $AP$, $A'Q$ is a rectangular hyperbola.

28. If two concentric rectangular hyperbolas be described, the axes of one being the asymptotes of the other, they will intersect at right angles.

29. If any chord $AP$ through the vertex be divided in $Q$, so that $AQ : QP :: AC^2 : BC^2$, and $QN$ be drawn to the foot of the ordinate $PN$, prove that a straight line drawn at right angles to $QN$ from $Q$ cuts the transverse axis in the same ratio.

30. Prove that the curve which trisects the arcs of all segments of a circle described upon a given base is an hyperbola.

31. If $SVs$, $TVt$ be two tangents cutting one asymptote in $S$, $T$, and the other in $s$, $t$, prove that

$$VS : Vs :: Vt : VT.$$

32. If from the exterior focus of an hyperbola a circle be described with radius equal to $BC$, and tangents be drawn to it from any point in the hyperbola, the line joining the points of contact will touch the circle described on the transverse axis as diameter.

33. Circles are drawn touching the straight line $AB$ in a fixed point $C$; and from the fixed points $A$, $B$ tangents are drawn to these circles. The locus of their intersection is an ellipse or hyperbola. Distinguish between the two cases.

34. $PP'$ is a double ordinate in an ellipse. $AP$, $A'P'$ are produced to meet in $Q$. Prove that the locus of $Q$ is an hyperbola with the same axes as the ellipse.

35. If the tangent at $P$ intersect the asymptotes in $L$ and $l$, and $PG$ be the normal at $P$, then the angle $LGl$ is a right angle.

36. If an ellipse, a parabola, and a hyperbola, have a common tangent, and the same curvature at the vertex, the ellipse will be entirely within the parabola, and the parabola entirely within the hyperbola.

37. The chord $RPP'R'$ of an hyperbola intersects the asymptotes in $R$ and $R'$. From the point $R$ a tangent $RQ$ is drawn meeting the hyperbola in $Q$. If $PH$, $QK$, $P'H'$ be drawn parallel to one asymptote meeting the other in the points $H$, $K$, $H'$; then $PH + P'H' = 2QK$.

38. If through $P$,$P'$ on an hyperbola lines be drawn parallel to the asymptotes forming a parallelogram, of which $PP'$ is one diagonal, the other diagonal will pass through the centre.

39. If $P$ be the middle point of a line $EF$ which moves so as to cut off a constant area from the corner of a rectangle, its locus is an equilateral hyperbola.

40. $PM$, $PN$ are drawn parallel to the asymptotes $CN$, $CM$, and an ellipse is constructed having $CN$, $CM$ for semi-conjugate diameters. If $CP$ cut the ellipse in $Q$, the tangents at $Q$ and $P$ to the ellipse and hyperbola are parallel.

41. If a circle be described through any point $P$ of a given hyperbola and the extremities $A$, $A'$ of the transverse axis, and $NP$ be produced to meet the circle in $Q$; prove that $Q$ traces out an hyperbola whose conjugate axis is a third proportional to the conjugate and transverse axes of the original hyperbola.

42. If lines be drawn from any point of a rectangular hyperbola to the extremities of a diameter, the difference between the angles which they make with the diameter will be equal to the angle which this diameter makes with its conjugate.

43. If between a rectangular hyperbola and its asymptotes any number of concentric elliptic quadrants be inscribed, the rectangle contained by their axes will be constant.

44. In the rectangular hyperbola if $CP$ be produced to $Q$ so that $PQ = CP$, and $QO$ be drawn at right angles to $CQ$ to intersect the normal in $O$, $O$ is the centre of curvature at $P$.

45. With two conjugate diameters of an ellipse as asymptotes a pair of conjugate hyperbolas are constructed; prove that if one hyperbola touch the ellipse the other will do so likewise; prove also that the diameters drawn through the points of contact are conjugate to each other.

46. If a pair of conjugate diameters of an ellipse when produced be asymptotes to an hyperbola, the points of the hyperbola at which a tangent to the hyperbola will also be a tangent to the ellipse, lie in an ellipse similar to the given one.

47. In the rectangular hyperbola the radius of curvature at $P$ is to the radius of curvature at $P'$ in the triplicate ratio of $CP$ to $CP'$.

48. $OP$, $OQ$ are tangents to an ellipse at $P$ and $Q$, and asymptotes to an hyperbola. Show that a pair of their common chords is parallel to $PQ$. One of these chords being $RS$, prove that if $PR$ touches the hyperbola at $P$, $QS$ touches it at $S$; also if $PS$, $QR$ meet in $U$, $OU$ bisects $PQ$.

49. The base of the triangle $ABC$ remains fixed, while the vertex $C$ moves in an equilateral hyperbola passing through $A$ and $B$. If $P$, $Q$ be the points in which $AC$, $BC$ meet the circle described on $AB$ as diameter, the intersection of $AQ$, $BP$ is on the other branch of the hyperbola.

# CHAPTER IV.

## THE SECTIONS OF THE CONE.

68. Def. If two indefinite straight lines *IOI′*, *DOD′*, intersect one another at a point *O*, and one of them *IOI′* remain fixed while the other *DOD′* revolves round it in such a manner that its inclination to *IOI′* is the same in all positions, the surface generated by *DOD′* will be a *Right Cone.*

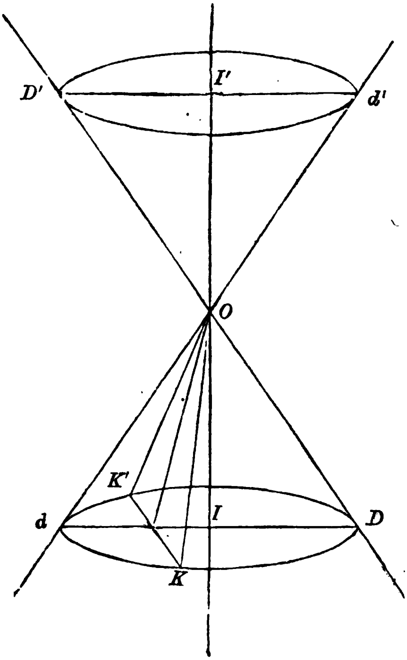
The line *IOI′* is called the *Axis*, and the point *O* the *Vertex* of the *Cone.*

It now remains for us to show (*see Introduction*) that the curve formed by the intersection of this surface with a plane is *in general* one of the three curves whose properties we have been investigating, and to consider under what circumstances it will be the Parabola, Ellipse, or Hyperbola.

If the cutting plane pass through the vertex of the cone as *KOK′*, and intersect the cone again *at all*, it will *in general* cut it in two straight lines as *OK*, *OK′*, which will represent two positions of the generating line.

The inclination of these lines to each other will depend upon the inclination of the cutting plane to the axis of the cone, and will be greatest when this plane passes through the axis, in which case it will be double the constant angle between the axis and the generating line.

If the cutting plane pass through a generating line *dod′* and be perpendicular to the plane containing this line and the axis, it will simply touch the cone along this line.



Should the cutting plane not pass through the vertex, and be at right angles to the axis of the cone, the section will evidently be a circle.

In any other case the section will, as we proceed to show, be a Parabola, Ellipse, or Hyperbola.

Whatever be the position of the cutting plane with respect to the cone, we can always suppose a plane drawn through the axis of the cone at right angles to it; and it will be convenient to have this latter plane represented by the plane of the paper as $DOd$. The cutting plane will therefore always be taken at right angles to the plane $DOd$ of the paper.

### Prop. I.

69. The curve formed by the intersection of the surface of a right cone with a plane (which neither passes through its vertex nor is at right angles to its axis) will be a *Parabola*, *Ellipse*, or *Hyperbola*, according as the inclination of the cutting plane to the axis of the cone is *equal to*, *greater*, or *less than* the constant angle which the generating line forms with the axis.

Let the plane of the paper represent the plane drawn through the axis $IOI'$ of the cone at right angles to the cutting plane; and let it intersect the surface of the cone in the two generating lines $OD$, $Od$.

Let the cutting plane intersect the surface of the cone in the curve $PA$, and the plane of the paper in the line $ANH$.

The curve will evidently be symmetrical with respect to this line.

On $AH$ take any point $N$, and through $N$ draw a plane perpendicular to the axis meeting the surface of the cone in the circle $RPr$, and the cutting plane in the line $PN$, which will be at right angles to the plane of the paper and to $AN$.

Let a sphere be inscribed in the cone touching the cone in the circle $EQe$ and the cutting plane in the point $S$, and let the plane $EQe$ intersect the cutting plane in the line $XM$, which will be at right angles to the plane of the paper, and therefore parallel to $PN$.

Draw $PM$ perpendicular to $XM$, and join $PS$, $PO$, and let $PO$ meet the circle $EQe$ in the point $Q$.*

Then since $PS$ and $PQ$ are both tangents to the sphere,

$$\therefore PS = PQ.$$

$$\text{But } PQ = RE,$$

$$\therefore PS = RE.$$

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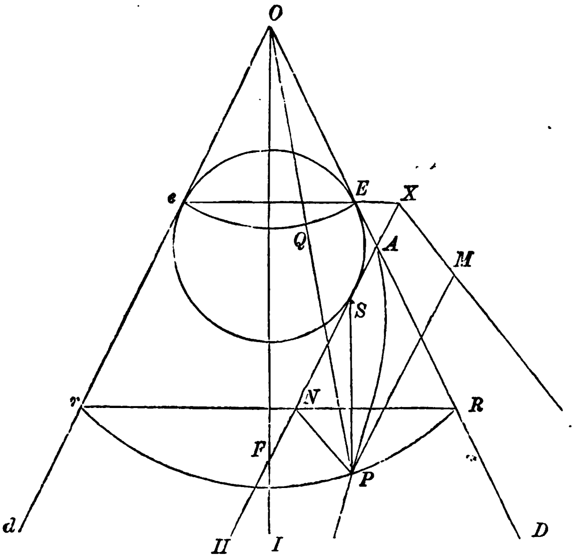
* N.B. In the figure, to avoid confusion, that part of the section which lies above the plane of the paper is alone represented.



But $RE : XN :: AE : AX$, (*Euclid*, VI. 2.)

and $AE = AS$,

$\therefore RE : XN :: AS : AX$,

$\therefore SP : PM :: AS : AX$,

$\therefore$ the curve $PA$ is either a *Parabola*, *Ellipse*, or *Hyperbola*, whose focus is $S$ and directrix $XM$.

Again, let $AH$ meet the axis $OI$ in $F$.

Then the angle $AFO$ will be the inclination of the cutting plane to the axis.

(1) Let the angle $AFO =$ the angle $FOd$;

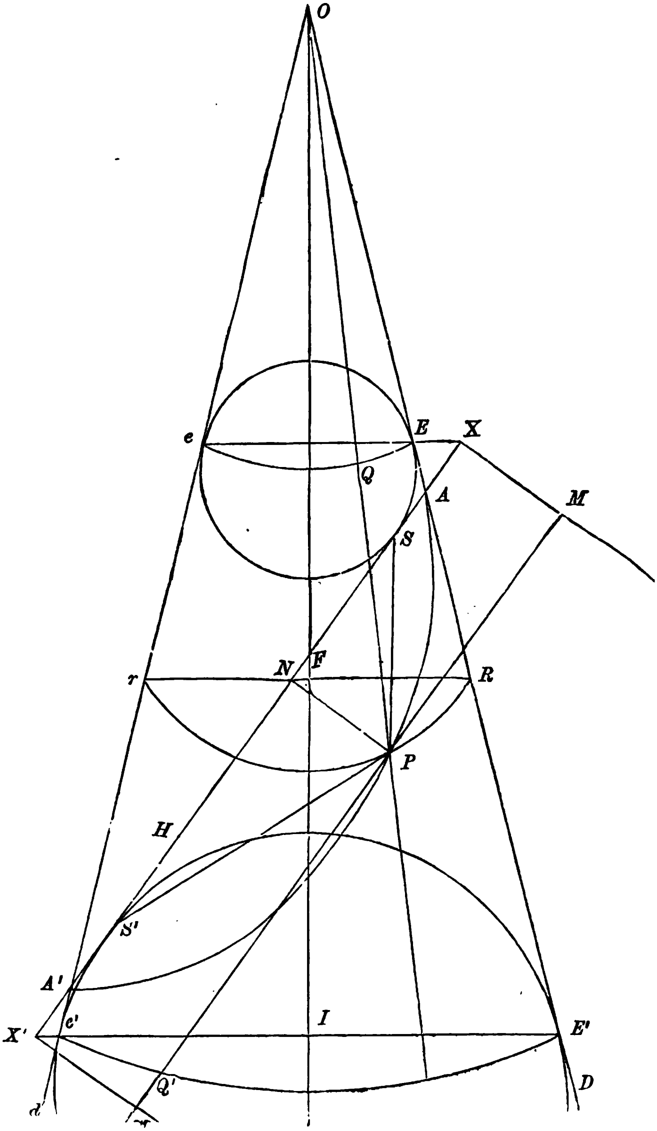
then $AH$ is parallel to $Od$,

$\therefore$ the angle $AXE =$ the angle $OeE =$ the angle $AEX$,

$\therefore AE = AX$,

$\therefore AS = AX$,

$\therefore$ the curve $AP$ will be a *Parabola*.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(2) Let the angle $AFO$ be $> FOd$; then

the complement $FXE$ is $<$ the complement $OEe$,

$\therefore$ the angle $AXE$ is $< AEX$,

$\therefore$ $AE$ is $< AX$,

or $AS$ is $< AX$,

$\therefore$ the curve $AP$ is an *Ellipse.*

Since the angles $HFO$, $FOd$ are together less than $HFO$, $OFA$, *i.e.* than two right angles, the lines $AH$ and $Oe$ may be produced to meet in $A'$.

If another sphere be described touching the cone in the circle $E'Q'e'$ and the cutting plane in the point $S'$; and the line $X'M'$ denote the intersection of the plane $E'Q'e'$ with the cutting plane, and $PM'$ be drawn at right angles to this line, it can easily be shown that

$$S'P : PM' :: S'A' : A'X'.$$

Hence $S'$ and $X'M'$ represent respectively the other focus and directrix of the ellipse.

Also if $BC$ be the semi-axis minor, and through the centre $C$ a line $UCU'$ be drawn parallel to $Ee$ meeting $OD$, $Od$ in $U$ and $U'$, then it is evident that

$$BC^2 = CU \cdot CU'.$$

(3) Let the angle $AFO$ be $< FOd$; then

the angle $AXE$ is $>$ the angle $AEX$,

$\therefore$ $AE$ is $> AX$,

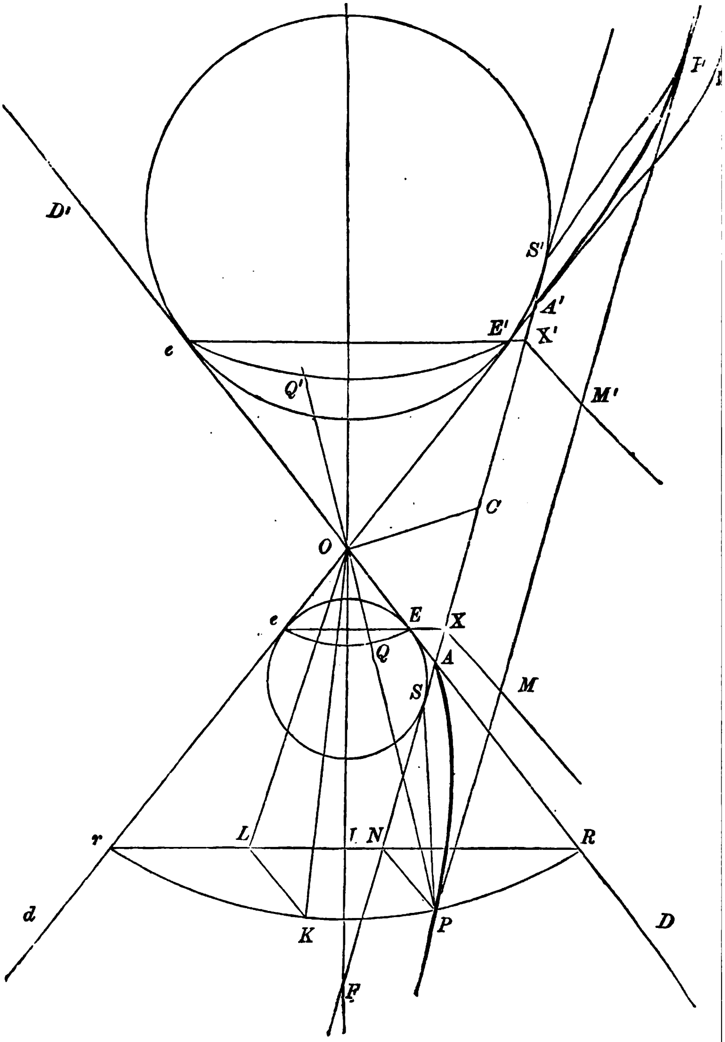
$\therefore$ $AS$ is $> AX$,

$\therefore$ the curve $PA$ is an *Hyperbola.*

Since the angles $AFO$, $FOd'$ are less than the two $FOd$, $FOd'$, *i.e.* than two right angles, the lines $FA$ and $dO$ may be produced to meet in $A'$.

In this case the cutting plane will intersect the other half of the cone, and if any point $P'$ be taken on this part of the curve, and $P'M$ be drawn at right angles to $XM$, it can be shown as before that

$$SP' : P'M :: S'A :: AX.$$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The intersection of the cutting plane therefore with this portion of the cone will be the other branch of the hyperbola.

Also if another sphere be described touching the upper portion of the cone in $E'Q'e'$, and the cutting plane in $S'$, and the line $X'M'$ denote the intersection of the plane $E'Q'e'$ with the cutting plane, and $P'M'$ be drawn at right angles to this line, it can be easily shown that

$$S'M' : P'M' :: S'A' : A'X'.$$

Hence, $S'$ and $X'M'$ will represent respectively the other focus and directrix of the hyperbola.

COR. 1. In this last case, *i.e.* when the section is an hyperbola, if a plane $OKL$ be drawn through the vertex of the cone parallel to the cutting plane, meeting the plane of the paper in the straight line $OL$, and the surface of the cone in the generating line $OK$; then

$$\begin{aligned} OL : OK &:: OL : OR, \\ &:: AN : AR, \\ &:: AX : AE, \quad (\textit{Euclid}, \text{VI. } 2.) \\ &:: AX : AS, \\ &:: CA : CS, \quad (\textit{Chap.} \text{ III. } \textit{Prop.} \text{ II.}) \end{aligned}$$

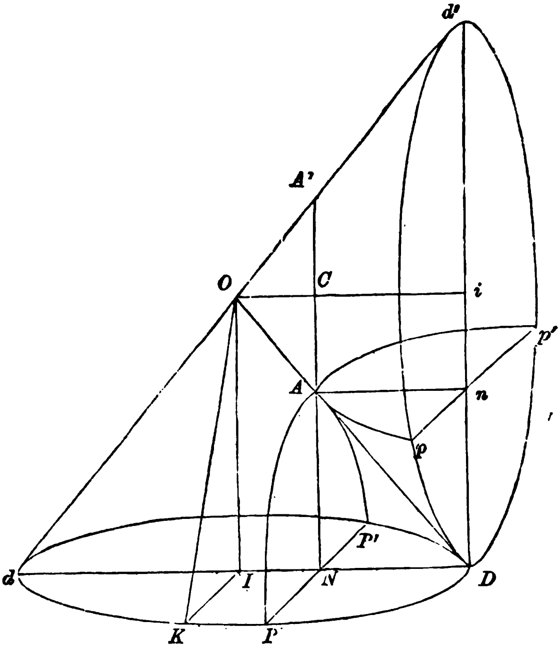
where $C$ is the middle point of $AA'$ and therefore the centre of the hyperbola.

$\therefore$ $KOL$ is half the angle between the asymptotes. (*Chap.* III. *Prop.* XVI.)

Again, if $BC$ be the conjugate semi-axis, and $CU'U$ be drawn parallel to $Rr$ meeting $OD'$, $Od'$ in $U$ and $U'$, then

$$\begin{aligned} \text{since } CU : AC &:: RL : OL, \\ \text{and } CU' : A'C &:: rL : OL, \\ \therefore CU \,.\, CU' : AC^2 &:: RL \,.\, rL : OL^2, \\ &:: KL^2 : OL^2; \\ \text{but } BC^2 : AC^2 &:: KL^2 : OI^2, \\ \therefore BC^2 &= CU \,.\, CU'. \end{aligned}$$

COR. 2. If the cutting plane is parallel to the axis $OL$ and $OI$ coincide.



In this case half the angle between the asymptotes of the hyperbolic section is equal to the constant angle $DOI$, and we can at once see that $OC$ is the semi-conjugate axis.

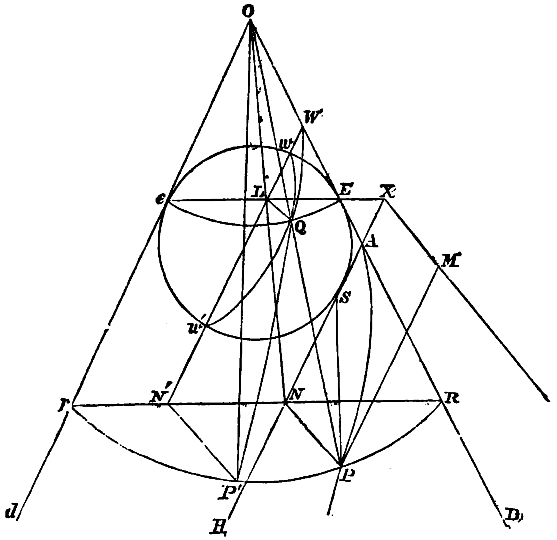
This affords a convenient method of obtaining a pair of conjugate hyperbolas.

Draw $Oi$ at right angles to $OI$ in the plane of the paper, and let another cone be formed by supposing $OD$ to revolve round $Oi$ in such a manner that the angle $DOi$ is the same in all positions, and equal to the complement of $DOI$.

Then if through any point $A$ on the common generating line $OD$ we draw two planes at right angles to the plane of the paper, and parallel respectively to $OI$ and $Oi$, they will cut the cones in two hyperbolas, whose semi-transverse axes will be respectively $AC$, $OC$, and whose semi-conjugate axes will be respectively $OC$, $AC$, and which therefore will be *conjugate* to each other.

70. As long as the cutting plane remains parallel to itself, it is evident that the ratio of $AE$ to $AX$, and therefore of $AS$ to $AX$ will be altered. Hence the sections made by planes inclined at the same angle to the axis of the cone will have the same eccentricity.*

71. Through any point $Q$ on the circle $EQe$ let a plane be drawn parallel to $ANP$, intersecting the plane of the paper in the straight line $WLN'$, the cone in the curve $WQP'$, and the planes of the circular sections $EQe$, $RPr$ in the ordinates $QL$, $P'N'$.



Then it is manifest that the curve $WQP'$ will touch the circle $wQw'$, formed by the intersection of the cutting plane with the sphere at the extremities of the ordinate $QL$ produced.

Join $OP'$ meeting $EQe$ in $Q'$; then

$$\begin{aligned} P'Q' : N'L &:: RE : NX, \\ &:: SA : AX. \end{aligned}$$

---

* The ratio of $SA : AX$, or of $CS : CA$, is called the eccentricity.

But $P'Q'$ is equal to the tangent drawn from $P'$ to the circle $wQw'$, and $N'L$ is equal to the perpendicular from $P'$ on the common ordinate of the circle $wQw'$ and the section $WQP'$.

Hence we have the following important property,—

If a circle touch a conic section in two points at the extremities of an ordinate, the ratio which the tangent drawn to the circle from any point on the curve bears to the perpendicular from the same point on the common chord is equal to the eccentricity of the conic section.

If the two points in which the circle touches the conic coincide, the circle becomes the circle of curvature at the vertex; and therefore the ratio which the tangent drawn from any point of a conic section to the circle of curvature at the vertex bears to the abscissa of the point, is constant and equal to the eccentricity of the curve.

## PROBLEMS ON THE SECTIONS OF THE CONE.

1. THE foci of all parabolic sections which can be cut from a given right cone lie upon the surface of another cone.

2. The foci of all elliptical sections of a given right cone, in which the ratio of $CA$ to $CS$ is the same, will lie on two other cones.

3. The extremities of the minor axes of the elliptical sections of a right cone made by parallel planes lie on two generating lines.

4. The latus rectum of a parabola cut from a given cone varies as the distance between the vertices of the cone and the parabola.

5. Under what conditions is it possible to cut an equilateral hyperbola from a given right cone?

6. Two cones whose vertical angles are supplementary are joined as in *Art.* 69, *Cor.* 2. Prove that the latera recta of the curves of section of the cones, whose axes are respectively $OI$ and $Oi$, made by planes parallel or perpendicular to the plane of the axes, are in the duplicate ratio of $Oi$ and $OI$.

## ADDITIONAL PROBLEMS.

1. Show that the part of the directrix of a parabola, intercepted between the perpendiculars on it from the extremities of any focal chord, subtends a right angle at the focus.

2. The locus of the foci of all parabolas touching the three sides of a triangle is a circle. Prove this, and give a geometrical construction for finding the centre.

3. A system of parabolas which always touch two given straight lines have their axes parallel; show that the locus of the foci is a straight line.

4. Prove that the locus at the foot of the perpendicular from the focus of a parabola on the normal is a parabola.

5. If $S$ be the focus of a parabola, which touches the sides $AB$, $AC$ of the triangle $ABC$ at the points $B$, $C$, and $O$ the centre of the circle described about the triangle; prove that the angle $OSA$ is a right angle.

6. From the focus of a parabola a straight line is drawn parallel to the tangent at any point $P$, meeting the diameter through $P$ in $V'$; show that the tangent drawn from $P$ to any circle passing through $V'$ is equal to one-half of the ordinate $QV$, $V$ being the second point in which the circle cuts the diameter through $P$.

7. $PSp$ is a focal chord, and upon $PS$ and $pS$ as diameters, circles are described; prove that the length of either of their common tangents is a mean proportional between $AS$ and $Pp$.

8. If $AQ$ be a chord of a parabola through the vertex $A$, and $QR$ be drawn perpendicular to $AQ$ to meet the axis in $R$; prove that $AR$ will be equal to the chord through the focus parallel to $AQ$.

9. The locus of the vertices of all parabolas, which have a common focus and a common tangent, is a circle.

10. Two parabolas have a common axis and vertex, and their concavities turned in opposite directions; the latus rectum of one is eight times that of the other; prove that the portion of a tangent to the former, intercepted between the common tangent and axis, is bisected by the latter.

11. $B$ is a point on a radius $OA$ of a circle, whose centre is $O$. On $OA$ produced a point $C$ is taken, such that $OB \cdot OC = OA^2$. If $P$ be any point on the circumference of this circle, $R$ the middle point of $BP$, and $Q$ the point of intersection of $AR$, $CP$; prove that the locus of $Q$ is a circle.

12. If from the middle point of a focal chord of a parabola two straight lines be drawn, one perpendicular to the chord meeting the axis in $G$, and the other perpendicular to the axis meeting it in $N$; show that $NG$ is constant.

13. A circle is drawn touching the axis of a parabola, the focal distance of a point $P$, and the diameter through $P$. Show that the locus of its centre is a parabola with vertex $S$, and latus rectum equal to $AS$.

14. If from the point of intersection of the directrix and axis of a parabola a chord $XPQ$ be drawn, cutting the parabola in $P$, $Q$; show that the rectangle contained by the ordinates of $PQ$ is equal to the square of one-half the latus rectum.

15. Find the locus of the centre of a circle which touches a given circle and a given straight line.

16. Given one point of contact of a parabola with three

tangents given in position, find the two other points of contact.

17. The triangle $ABC$ circumscribes a parabola whose focus is $S$. Through $A$, $B$, $C$, lines are drawn perpendicular respectively to $SA$, $SB$, $SC$. Show that these lines pass through one point.

18. From the focus a line is drawn parallel to the tangent at $P$, meeting the parabola at $Q$. $QN$ is an ordinate, and the tangents at $P$ and $Q$ meet the axis in $T$ and $T'$. Prove that $SN^2 = 4AT \, . \, AT'$, and that if the diameter at $P$ meet $SQ$ in $E$, the locus of $E$ is a parabola, whose latus rectum is half that of the given parabola.

19. $P$ is any point in a parabola; through $S$ a line is drawn at right angles to the axis, meeting the chord $AP$ or $AP$ produced in $R$. Prove that $SK \, . \, SR = 2AS \, . \, SY$, where $SY$ is the perpendicular on the tangent, and $SK$ on the normal.

20. From the focus $S$ of a parabola $SK$ is drawn, making a given angle with the tangent at $P$. Show that the locus of $K$ is that tangent to the parabola which makes with the axis an angle equal to the given angle.

21. $PSQ$ is a focal chord of a parabola, $AP'$ a parallel chord meeting the latus rectum in $Q'$; prove that $AP' \, . \, AQ = SP \, . \, SQ$.

22. The circle of curvature at any point of a parabola whose abscissa is $AN$ cuts the axis in $U$ and $U'$. Prove that $AU \, . \, AU' = 3\,AN^2$.

23. $AB$ is a diameter of a circle. From any point $Q$ in the circumference a tangent $QP$ is drawn, and from $P$ a perpendicular $PN$ is let fall upon $AB$. Show that if $P$ be always taken so that $QP$ is equal to $AN$, the locus of $P$ will be a parabola.

24. If a tangent be drawn from any point of a parabola to the circle of curvature at the vertex, the length of the tangent

will be equal to the abscissa of the point measured along the axis.

25. To two parabolas which have a common focus and axis two tangents are drawn at right angles; the locus of their intersection is a straight line parallel to the directrices.

26. If any three tangents be drawn to a parabola, the circle described about the triangle so formed will pass through the focus, and the perpendiculars from the angles on the opposite sides intersect in the directrix.

27. A parabola touches one side of a triangle in its middle point, and the other two sides produced. Prove that the perpendiculars drawn from the angles of the triangle upon any tangent to the parabola are in harmonical progression.

28. Two equal parabolas have the same axis and vertex, but are turned in opposite directions; chords of one parabola are tangents to the other. Show that the locus of the middle point of the chords is a parabola whose latus rectum is one-third of that of the given parabola.

29. Two equal parabolas have the same focus, and their axes are at right angles to each other, and a normal to one of them is perpendicular to a normal of the other; prove that the locus of the intersection of such lines is a parabola.

30. Show that in every ellipse there are two equal conjugate diameters, coinciding in direction with the diagonals of the rectangle, which touches the ellipse at the extremities of the axes.

31. If a circle be described through the two foci of an ellipse, cutting the ellipse, show that the angle between the tangents to this circle, and to the ellipse at either point of intersection, is equal to the inclination of the normal to the ellipse to the axis minor.

32. The points in which the tangents at the extremities of the transverse axis of an ellipse are cut by the tangent at any

point of the curve, are joined one with each focus; prove that the point of intersection of the joining lines lies in the normal at the point.

33. The external angle between any two tangents to an ellipse is equal to the semi-sum of the angles which the chord joining the points of contact subtends at the foci.

34. The tangent to an ellipse at any point $P$ is cut by any two conjugate diameters in $T, t$, and the points $T, t$, are joined with the foci $S, S'$ respectively; prove that the triangles $SPT$, $S'Pt$ are similar to each other.

35. $P$ is any point on a fixed circle, the centre of which is $O$; $E$ is a fixed point without the circle; an ellipse is described with centre $O$ and area constant so as always to touch $EP$ at $P$. Find the locus of the extremities of the diameter conjugate to $OP$.

36. The normal at any point $P$ of an ellipse cuts the axes in $G, g$; prove that if any circle be described passing through $G, g$, the tangent to it from $P$ is equal to $CD$.

37. Given a focus, a tangent, and the eccentricity of a conic section; prove that the locus of the centre is a circle.

38. A straight line is drawn through a given point $C$ within a circle to cut it in $P, P'$. If $p$ is taken in it such that $Cp^2 = CP \cdot CP'$, find the locus of $p$.

39. In the ellipse $PY \cdot PY' : PN^2 :: CS^2 : BC^2$ and $SY \cdot CD = SP \cdot BC$.

40. Show that if the distance between the foci of the ellipse be greater than the length of its axis minor, there will be four positions of the tangent, for which the area of the triangle, included between it and the straight lines drawn from the centre of the curve to the feet of the perpendiculars from the foci on the tangent, will be the greatest possible.

41. Two conjugate diameters of an ellipse are cut by the tangent at any point $P$ in $M$, $N$; prove that the area of the triangle $CPM$ varies inversely as that of the triangle $CPN$.

42. Circles are described on $SY$, $S'Y'$ as diameters, cutting $SP$, $S'P$ respectively in $Q$, $Q'$. Prove that $SQ \cdot S'P = SP \cdot S'Q' = BC^2$.

43. $PSP'$, $pSp'$ are any two focal chords of a conic section, $P$ and $p$ being on the same side of the axis; prove that $Pp$, $P'p'$ meet on the directrix.

44. Prove that an ellipse can be inscribed in any parallelogram so as to touch the middle points of the four sides; and show that this ellipse is the greatest of all inscribed ellipses.

45. If from any point on the exterior of two concentric, similar, and similarly placed ellipses, two tangents be drawn to the interior ellipse which also intersect the exterior; show that the distance between the points of intersection will be double of that between the points of contact.

46. The tangent at any point $P$ in an ellipse, of which $S$ and $H$ are the foci, meets the axis major in $T$, and $TQR$ bisects $HP$ in $Q$, and meets $SP$ in $R$; prove that $PR$ is one-fourth of the chord of curvature at $P$ through $S$.

47. Prove that the distance between the two points on the circumference of an ellipse at which a given chord, not passing through the centre, subtends the greatest and least angles, is equal to the diameter which bisects that chord.

48. From any point on the auxiliary circle chords are drawn through the foci of an ellipse, and straight lines join the extremities of the chords with the extremity of the diameter passing through the point; prove that these lines will touch the ellipse.

49. A quadrilateral circumscribes an ellipse. Prove that either pair of opposite sides subtends supplementary angles at either focus.

50. Two tangents to an ellipse intersect at right angles; show that the straight line joining their point of intersection with the point of intersection of the normals at the points of contact passes through the centre.

51. $P$, $Q$ are points in two confocal ellipses, at which the line joining the common foci subtends equal angles; prove that the tangents at $P$, $Q$ are inclined to an angle which is equal to the angle subtended by $PQ$ at either focus.

52. Tangents to an ellipse are drawn from any point in a circle through the foci; prove that the lines bisecting the angle between the tangents all pass through a fixed point.

53. If the ordinate at $P$ meet the auxiliary circle in $Q$, and $CQ$ meet the ellipse in $R$, then $CR$ is equal to the perpendicular on the tangent at $P$ from $C$.

54. If $P$ be a point such that $SP$, $S'P$ are perpendicular; prove that $CD^2 = 2 \,.\, BC^2$.

55. If circles be described to the triangle $SPS'$ opposite to the angles $S$ and $S'$; prove that the rectangle contained by their radii is equal to $BC^2$.

56. The circle of curvature at any point $P$ of an ellipse meets the focal distances in $R$, $R'$; $SU$ is a tangent to the circle.

Prove that $SU^2 : SP^2 :: 2 \,.\, SP - 3 \,.\, AC : AC$,

and if $RR'$ passes through the centre of the circle of curvature, $CP = CS$. Determine the limits of possibility in both cases.

57. A straight line is drawn from the centre of an ellipse meeting the ellipse in $P$, the circle on the major axis in $Q$, and the tangent at the vertex in $T$. Prove that as $CT$ approaches and ultimately coincides with the semi-major axis, $TP$ and $QT$ are ultimately in the duplicate ratio of the axes.

58. A straight line is drawn through the focus $S$ of an ellipse meeting the two tangents at right angles to it in $Y$ and $Z$, the diameter parallel to these tangents in $L$, and the directrix in $M$; prove that

$$SL : SY :: SZ : SM.$$

59. If any equilateral triangle $PQR$ be described in the auxiliary circle of an ellipse, and the ordinates to $P$, $Q$, $R$ meet the ellipse in $P'$, $Q'$, $R'$; the circles of curvature at $P'$, $Q'$, $R'$, meet in one point lying on the ellipse.

60. From a point $T$ two tangents $TP$, $TQ$ are drawn to an ellipse. Show that a circle with $T$ as centre can be described so as to touch $SP$, $S'P$, $SQ$, $S'Q$.

61. If the normal at $P$ meet the axis minor in $g$, and if the tangent at $P$ meet the tangent at the vertex $A$ in $V$; show that

$$Sg : SC :: PV : VA.$$

62. If a circle passing through $Y$ and $Y$ touch the major axis in $Q$, and that diameter of the circle which passes through $Q$ meet the tangent in $P$; show that $PR = BC$. (*See fig. Prop.* XV.)

63. If $PG$ the normal at $P$ cut the major axis in $G$, and if $DR$, $PN$ be the ordinates of $D$ and $P$, prove that the triangles $PGN$, $DRC$ are similar; and thence deduce that $PG$ bears a constant ratio to $CD$.

64. The tangent at a point $P$ of an ellipse meets the tangents at the vertices in $V$, $V'$. On $VV'$ as diameter, a circle is described which intersects the ellipse in $Q$, $Q'$; show that the ordinate of $Q$ is to the ordinate of $P$ as $BC$ to $BC + CD$ where $CD$ is conjugate to $CP$.

65. $PCP'$ is any diameter of an ellipse; the tangents at any two points $E$ and $E'$ intersect in $F$; $PE'$, $P'E$ intersect in $G$. Show that $FG$ is parallel to the diameter conjugate to $PCP'$.

66. If $P$ be any point on an ellipse, and with $P$ as centre and the semi-axis minor as radius a circle be described; prove that if $PG$ be the normal, a circle described on $CG$ as diameter will cut the first circle at right angles.

67. $ABC$ is an isosceles triangle having $AB = AC$. $BD$, $BE$ drawn on opposite sides of $BC$, and equally inclined to it, meet $AC$ in $D$ and $E$.

If an ellipse be described about $BDE$ having its minor axis parallel to $BC$; then $AB$ will be a tangent to the ellipse.

68. If $AQ$ be drawn from one of the vertices of an ellipse perpendicular to the tangent at any point $P$; prove that the locus of the point of intersection of $PS$ and $QA$ produced will be a circle.

69. If $Y$, $Y'$ be the feet of the perpendiculars from the foci of an ellipse on the tangent at $P$; prove that the circle circumscribed about the triangle $YNY'$ will pass through $C$.

70. Prove that the angle between the tangents to the auxiliary circle at $Y$, $Y'$ is the supplement of the angle $SPS'$.

71. $P$ is any point on an ellipse; $PM$, $PN$ perpendicular to the axes meet respectively, when produced, the circles described on the axes as diameter in the points $Q$, $Q'$. Show that $QQ'$ passes through the centre.

72. Assuming that the greatest triangle which can be inscribed in a circle is equilateral, prove by the method of projection, that the greatest triangle which can be inscribed in an ellipse has one of its sides bisected by a diameter of the ellipse, and the others cut in points of bisection by the conjugate diameter.

73. $PQ$ is a chord of an ellipse, normal at $P$, $LCL'$ the diameter bisecting it. Show that $PQ$ bisects the angle $LPL'$ and that $LP + L'P$ is constant.

74. A tangent to an ellipse at a point $P$ intersects a fixed tangent in $T$; if through the focus a line be drawn perpendicular to $ST$ meeting the tangent to $P$ in $Q$; show that the locus of $Q$ is a straight line touching the ellipse.

75. In an ellipse if a line be drawn through the focus making a constant angle with the tangent; prove that the locus of the point of intersection with the tangent is a circle.

76. Any chord $PP'$ of an ellipse is produced to a point $Q$, such that $P'Q$ is equal to half the diameter parallel to $PP'$, and $QR'R$ is drawn through the centre to meet the ellipse in $R$, $R'$; show that the area $PCR$ is three times the area $P'CR'$.

77. In an ellipse, $L$ is the extremity of the latus rectum, and $CD$ conjugate to $CL$. If a circle be described with centre $C$ and passing through $B$, and a line be drawn through $D$ parallel to the major axis, the portion of this line which lies within the circle will be equal to the latus rectum.

78. If $P$ be any point in an ellipse, and $K$ the point in which a normal at $P$ intersects a line at right angles to it through $S'$, $E$ the point of intersection of $SP$, and the diameter conjugate to $CP$, and if $EK$ and $CK$ be joined, each of the figures $SCKE$, $S'CEK$ will be a parallelogram.

79. If $T$ be a point on the axis $AA'$ produced, and $PN$ the ordinate of the point where the tangent from $T$ touches the ellipse; prove that

$$AN \,.\, A'N : AT \,.\, A'T :: CN : CT.$$

80. Given in an ellipse a focus and two tangents; prove that the locus of the other focus is a straight line.

81. A focus, a tangent, and the axis major being given, prove that the locus of the other focus is a circle.

82. A focus, a tangent, and the axis minor being given, prove that the locus of the other focus is a straight line.

83. An ellipse touches a fixed ellipse and has a common focus with it; if the major axis be fixed, the locus of the other focus is a circle; if the minor axis be fixed, the locus is an ellipse.

84. An ellipse and a parabola have a common focus. Prove that the ellipse either intersects the parabola in two points, and has two common tangents with it, or else does not cut it.

85. If in the ellipse a focus, a point, and the axis minor be given, the locus of the other focus is a parabola.

86. If at the extremities $P$, $Q$ of any two diameters $CP$, $CQ$ of an ellipse, two tangents $Pp$, $Qp$ be drawn cutting each other in $T$, and the diameter produced in $p$ and $q$, then the area of the triangles $TQp$, $TPq$ are equal.

87. If a straight line $CN$ be drawn from the centre to bisect that chord of the circle of curvature at any point $P$ of an ellipse, which is common to the ellipse and circle, and if it be produced to cut the ellipse in $Q$, and the tangent in $T$; prove that $CP = CQ$, and that each is a mean proportional between $CN$ and $CT$.

88. An ellipse is described so as to touch the three sides of a triangle; prove that if one of its foci move along the circumference of a circle passing through two of the angular points of the triangle, the other will move along the circumference of another circle, passing through the same two angular points. Prove also that if one of these circles pass through the centre of the circle inscribed in the triangle, the two circles will coincide.

89. A triangle is described about an ellipse, so that the extremities of one of its sides lie in an ellipse, confocal with the given one; prove that the line bisecting the opposite angle passes through the pole of that side with respect to the outer ellipse.

90. Prove the following construction for a pair of tangents from any external point $T$ to an ellipse of which the centre is $C$. Join $CT$; let $TPCP'T$, a similar and similarly situated ellipse, be drawn, of which $CT$ is a diameter, and $P$, $P'$ its points of intersection, with the given ellipse; $TP$, $TP'$ will be tangents to the given ellipse.

91. The locus of the foci of all ellipses inscribed in the same parallelogram is a rectangular hyperbola. Prove this, and give a geometrical construction for finding the asymptotes.

92. $AC$ is a fixed diameter of a circle, $ABCD$ a quadrilateral figure inscribed in the circle; prove that if the angles $BAC$, $DAC$ be complementary, the locus of the intersection of $BA$, $CD$ will be an hyperbola.

93. Prove that a circle can be described so as to touch the four straight lines drawn from the foci of an hyperbola to any two points on the same branch of the curve.

94. Any three diameters of an ellipse, $LL'$, $MM'$, $NN'$, being taken, a circumscribing parallelogram $RTUV$ touches the ellipse at $L$, $L'$, $M$, $M'$. Show that a conic section can be described through the points $R$, $T$, $U$, $V$, $N$, $N'$, which will be an hyperbola whose asymptotes are the lines forming in the ellipse the diameters conjugate to $NN'$ and to the other common chord of the ellipse and hyperbola.

95. On opposite angles of any chord of a rectangular hyperbola are described equal segments of circles; show that the four points in which the circles to which these segments belong again meet the hyperbola, are the angular points of a parallelogram.

96. A triangle is inscribed in a rectangular hyperbola: prove that the circle described through the middle points of the sides of the triangle passes through the centre of the hyperbola.

97. $ACB$ is an isosceles triangle; $AB$ the base, and $D$ any point in $CB$ or $CB$ produced: if $BZ$ be drawn parallel to $AD$, meeting $CA$ or $CA$ produced in $Z$, prove that the middle point of $DZ$ will be in an hyperbola whose asymptotes are $CA$, $CB$.

98. An ellipse and hyperbola are described so that the foci of each are at the extremities of the transverse axis of the other; prove that the tangents at their points of intersection meet the conjugate axis in points equidistant from the centre.

99. In a rectangular hyperbola, $PK$, $PL$ are drawn at right angles to $A'P$, $AP$ respectively to meet the transverse axis in $K$ and $L$; prove that $PK$ is equal to $AP$ and $KL$ to $AA'$, and the normal at $P$ bisects $KL$.

100. In a rectangular hyperbola $PC$ is a fixed diameter, $Q$ any point on the curve; show that the angles $QPC$, $QCP$ differ by a constant angle.

101. If the tangent at any point $P$ of an hyperbola cut an asymptote in $T$, and if $SP$ cut the same asymptote in $Q$, then $SQ = QT$.

102. If a given point be the focus of any hyperbola, passing through a given point and touching a given straight line, prove that the locus of the other focus is an arc of a fixed hyperbola.

103. At any $P$ of an hyperbola a tangent is drawn, and $PQ$ is taken on it in a constant ratio to $CD$; prove that the locus of $Q$ is an hyperbola.

104. In an hyperbola, supposing the two asymptotes and one point of the curve be given in position, show how to construct the curve; and find the position of the foci.

105. If $A$, $D$ be two fixed points, and the angle $PAD$ always exceed $PDA$ by a given angle; find the locus of $P$, and the position of the transverse axis and asymptote.

106. From the middle point $D$ of the base $AB$ of the triangle $ABC$ a straight line $EDE'$ is drawn, making a given angle with $AB$, and the points $E$, $E'$ are taken so that $ED = E'D = \frac{1}{2} AB$. If $CA$, $CB$ take all possible positions consistent with the condition that the difference of the angles $CAB$, $CBA$ is equal to $EDA$; prove that the point $C$ will trace out a rectangular hyperbola of which $AB$, $E'E$ are conjugate diameters.

107. In the rectangular hyperbola, prove that the triangle, formed by the tangent at any point and its intercepts on the axes, is similar to the triangle formed by the straight line joining that point with the centre, and the abscissa and semi-ordinate of the point.

108. Tangents are drawn to an hyperbola, and the portion of each tangent intercepted by the asymptotes is divided in a constant ratio; prove that the locus of the point of section is an hyperbola.

109. Show that the point of trisection of a series of conterminous circular arcs lie on branches of two hyperbolas, and determine the distance between their centres.

110. From a point $R$ on one asymptote $RE$ is drawn touching the hyperbola in $E$, and $ET$, $EV$ are drawn through $E$, parallel to the asymptotes, cutting a diameter in $T$ and $V$; $RV$ is joined, cutting the hyperbola in $P$, $p$: show that $TP$, $Tp$ touch the hyperbola.

111. Given in the ellipse a focus and two points, the locus of the other focus is an hyperbola.

112. If a rectangular hyperbola passes through three given points, the locus of its centre is a circle, which passes through the middle points of the lines joining the three given points.

113. If the tangent at $P$ meet one asymptote in $T$, and a line $TQ$ be drawn parallel to the other asymptote to meet the curve in $Q$; prove that if $PQ$ be joined and produced both ways to meet the asymptotes in $R$ and $R'$, $RR'$ will be trisected at the points $P$ and $Q$.

114. If two concentric rectangular hyperbolas have a common tangent, the lines joining their points of intersection to their respective points of contact with the common tangent will subtend equal angles at their common centres.

115. If $TP$, $TQ$ be two tangents drawn from any point $T$ to touch a conic in $P$ and $Q$, and if $S$ and $S'$ be the foci, then

$$ST^2 : S'T^2 :: SP \, . \, SQ : S'P \, . \, S'Q.$$

116. The circle of curvature at the vertex of a conic meets the axis again in $D$; and a tangent is drawn to the circle at $D$: if two tangents be drawn to the circle from any point in the conic they will intercept between them a constant length of the former tangent.

117. If the lines which bisect the angles between pairs of tangents to an ellipse be parallel to a fixed straight line, prove that the locus of the points of intersection of the tangents will be a rectangular hyperbola.

118. An hyperbola, of given eccentricity, always passes through two given points; if one of its asymptotes always pass through a third given point in the same straight line with these, prove that the locus of the centre of the hyperbola will be a circle.

119. $A$, $P$ and $B$, $Q$ are points taken respectively in two parallel straight lines, $A$ and $B$ being fixed, and $P$, $Q$ variable. Prove that if the rectangle $APBQ$ be constant, the line $PQ$ will always touch a fixed ellipse or a fixed hyperbola, according as $P$ and $Q$ are on the same or opposite sides of $AB$.

120. If two plane sections of a right cone be taken, having the same directrix, the foci corresponding to that directrix lie on a straight line which passes through the vertex.

121. Give a geometrical construction by which a cone may be cut so that the section may be an ellipse of given eccentricity.

122. Given a right cone and a point within it, there are but two sections which have this point for focus; and the planes of these sections make equal angles with the straight line joining the given point and the vertex of the cone.

123. If the curve formed by the intersection of any plane with a cone be projected upon a plane perpendicular to the axis, prove that the curve of projection will be a conic section having its focus at the point in which the axis meets the plane of projection.

124. If $F$ be the point where the major axis of an elliptic section meets the axis of the cone, and $C$ be the centre of the section; prove that

$$CF : CS :: AA' : AO + A'O,$$

$O$ being the vertex of the cone.

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## SECOND SERIES.

1. If $OQ$, $OQ'$ be tangents to a parabola, and $OV$ drawn parallel to the axis meet the directrix in $K$ and $QQ'$ in $V$, and $QQ'$ meet the axis in $N$, $OKNS$ shall be a parallelogram.

2. $G$ is the foot of a normal at a point $P$ of a parabola, $Q$ is the middle point of $SG$, and $X$ is the foot of the directrix, prove that

$$QX^2 - QP^2 = 4\,AS^2.$$

3. Through any point $O$ tangents are drawn to a parabola; and through $O$ straight lines are drawn parallel to the normals at the points of contact, prove that one diagonal of the parallelogram so formed passes through the focus.

4. Two parabolas have their foci coincident, prove that the common chord passes through the intersection of the directrices, and that they cut one another at angles which are half the angles between the axes.

5. A circle is drawn through the point of intersection of two given straight lines and through another given point, prove that the straight line joining the points, where the circle again meets the two given straight lines, touches a fixed parabola.

6. Common tangents are drawn to two parabolas which have a common directrix and intersect in $P$, $Q$; prove that the chords joining the points of contact in each parabola are parallel to $PQ$; and the part of each tangent between its points of contact with the two curves is bisected by $PQ$ produced.

7. The tangents at two points $Q$, $Q'$ in the parabola meet the tangent at $P$ in $R$, $R'$ respectively, and the diameter through their point of intersection $T$ meets it in $K$; prove that $PR = KR'$ and that if $QM$, $Q'M'$, $TN$ be the ordinates of $Q$, $Q'$, $T$ respectively to the diameter through $P$, $PN$ is a mean proportional between $PM$ and $PM'$.

8. $P$ and $p$ are points on a parabola on the same side of the axis and $PN$, $pn$ the ordinates; the normals at $P$ and $p$ intersect in $Q$; prove that the distance of $Q$ from the axis

$$= \frac{2PN \,.\, pn(PN + pn)}{(\text{latus rectum})^2}.$$

Deduce the value of the radius of curvature.

9. Two equal parabolas have a common focus, and from any point on the common tangent another tangent is drawn to each; prove that these tangents are equidistant from the common focus.

10. From an external point $O$ two tangents are drawn to a parabola, and from the points where they meet the directrix two other tangents are drawn meeting the tangents from $O$ at $A$ and $B$.

Prove that $AB$ passes through the focus $S$, and that $OS$ is at right angles to $AB$.

11. Given two tangent lines to a parabola and the focus, show how to determine the curve.

12. Inscribe in a given parabola a triangle having its sides parallel to those of a given triangle.

13. Normals at $P$, $p$ the extremities of a focal chord of a parabola meet the axis in $G$ and $g$. The tangents meet in $T$. Prove that the circles about the triangles $SPG$, $Spg$ intersect on $TS$ produced at a point $R$ such that $S$ bisects $RT$.

14. A circle and parabola touch one another at both ends of a double ordinate to the parabola, prove that the latus rectum is a third proportional to the parts into which the abscissa of the points of contact is divided by the circle either internally or externally.

15. $PQ$ is a chord of a parabola normal at $P$; $QR$ is drawn parallel to the axis to meet the double ordinate $PP'$ produced in $R$; then the rectangle contained by $PP'$ and $P'R$ is constant.

16. $OP$, $OQ$ are two tangents to a parabola, and $OM$ is drawn perpendicular to the axis; if $R$ be the point where $PQ$ cuts the axis, $MR$ is bisected by the vertex.

17. A parabola, whose focus is $S$, touches the three sides of a triangle $ABC$, bisecting the base $BC$ in $D$; prove that $AS$ is a fourth proportional to $AD$, $AB$, and $AC$.

18. If the chord of contact be normal at one end, the tangent at the other is bisected by the perpendicular through the focus to the line joining the focus to the external point.

19. Two parabolas have a common focus, and from any point on their common tangent are drawn two other tangents to the parabolas; prove that the angle between them is equal to the angle between the axes of the parabolas.

20. If tangents be let fall on any tangent to a parabola from two given points on the axis equidistant from the focus, the difference of their squares is constant.

21. If a quadrilateral be inscribed in a circle, one of the three diagonals of the quadrilateral passes through the focus of the parabola which touches its four sides.

22. A chord of a parabola is drawn parallel to a given straight line, and on this chord as diameter a circle is described; prove that the distance between the middle point of this chord, and of the chord joining the other two points of intersection of the circle and parabola will be of constant length.

23. Through any point $P$ of an ellipse a line is drawn perpendicular to the radius vector $CP$ meeting the auxiliary circle in $R$, $R'$: prove that $RR'$ is equal to the difference of the focal distances of the extremity of the diameter conjugate to $CP$.

24. An ellipse and parabola whose axes are parallel, have the same curvature at a point $P$ and cut one another in $Q$; if the tangent at $P$ meet the axis of the parabola in $T$ prove that $PQ$ is four times $PT$.

25. A triangle is inscribed in an ellipse so that each side is parallel to the tangent at the opposite angle: prove that the sum of the squares on the sides is to the sum of the squares on the axes as nine to eight.

26. In an ellipse the perimeter of the quadrilateral formed by the tangent, the perpendiculars from the foci and the transverse axis, will be the greatest possible, when the focal distances of the point of contact are at right angles to each other.

27. Two given ellipses in the same plane have a common focus, and one revolves about the common focus, while the other remains fixed; prove that the locus of the point of intersection of their common tangents is a circle.

28. Perpendiculars $SY$, $S'Y'$ are drawn from the foci upon a pair of tangents $TY$, $TY'$; prove that the angles $STY$, $S'TY'$ are equal or supplementary to the angles at the base of the triangle formed by joining $Y$, $Y'$ to the centre of the ellipse.

29. With the centre of perpendiculars of a triangle as centre, are described two ellipses, one about the triangle, and the other touching its sides: prove that these ellipses are similar and their homologous axes at right angles.

30. Through a point $P$ of an ellipse a line $PDE$ is drawn cutting the axes so that the segments $PD$ and $PE$ are equal to the two semi-axes respectively; perpendiculars to the axes through $D$ and $E$ intersect in $O$; prove that $PO$ is a normal.

31. If the focal distance $SP$ of an ellipse meet the conjugate diameter in $E$; then the difference of the squares on $CP$ and $SE$ will be constant.

32. The chords of curvature through any two points of an ellipse in the direction of the line joining them are in the same ratio as the squares of the diameters parallel to the tangents at the points.

33. From the extremities of the diameter of an ellipse, perpendicular to one of the equi-conjugate diameters, chords are drawn parallel to the other. Prove that these chords are normal to the ellipse.

34. An ellipse of given semi-axes touches three sides of a given rectangle; find its centre and foci.

35. If $P$, $Q$ be any two points on a fixed ellipse, whose foci are $S$, $H$ and if $SP$, $HQ$ intersect within the ellipse at $R$, prove that two ellipses can be drawn touching each other at $R$, the one having $S$ for focus and touching the given ellipse at $Q$, the other having $H$ for focus and touching the given ellipse at $P$.

If the major axis of one of the variable ellipses be given find the loci of their other foci, and of their point of contact.

36. Find the positions of the foci and directrices of an ellipse, which touches at two given points $P$, $Q$, two given straight lines $PO$, $QO$, and has one focus on the line $PQ$, the angle $POQ$ being less than a right angle.

37. If $Pp$ be drawn parallel to the transverse axis of a given ellipse, meeting the ellipse in $P, p$, and the circle whose diameter is the conjugate axis in $R$, $r$, then shall

$$Pp : Rr :: QQ' : PP'.$$

38. Through any point $P$ of an ellipse are drawn straight lines $APQ$; $A'PR$ meeting the auxiliary circle in $Q$, $R$, and ordinates $Qq$, $Rr$ are drawn to the transverse axis; prove that, $L$ being an extremity of the latus rectum

$$Aq \,.\, A'r : Ar \,.\, A'q :: AC^2 : SL^2.$$

39. Two ellipses whose axes are equal, each to each, are placed in the same plane with their centres coincident, and axes inclined to each other. Draw their common tangents.

40. If $S$, $S'$ be the two foci of an ellipse; and $SY$ the perpendicular from $S$ upon any tangent, prove that $S'Y$ will bisect the corresponding normal.

41. $OR$ is a diagonal of the parallelogram of which $OQ$, $OQ'$, tangents to an ellipse, are adjacent sides: prove that if $R$ be in the ellipse, $O$ will lie on a similar and similarly situated concentric ellipse.

42. A circle passes through a focus of an ellipse, has its centre on the major axis of the ellipse, and touches the ellipse: shew that the straight line from the focus to the point of contact is equal to the latus rectum.

43. If the focus of an ellipse be joined with the point where the tangent at the nearer vertex intersects any other tangent, and perpendiculars be drawn from the other focus on the joining line, and the last mentioned tangent, prove that the distance between the feet of these perpendiculars is equal to the distance from either focus to the remoter vertex.

44. A parallelogram is described about an ellipse; if two of its angular points lie on the directrices, the other two will lie on the auxiliary circle.

45. $PS$ is the focal chord of a point on an ellipse; $CR$ is a radius of the auxiliary circle parallel to $PS$, and drawn in the direction from $P$ to $S$; $SQ$ is a perpendicular on $CR$; shew that the rectangle contained by $SP$ and $QR$ is equal to the square on half the minor axis.

46. From the extremity $P$ of the diameter $PQ$ of an ellipse the tangent $TPT'$ is drawn meeting two conjugate diameters in $T$, $T'$. From $PQ$ the lines $PR$, $QR$ are drawn parallel to the same conjugate diameters. Prove that the rectangle under the semi-axes of the ellipse is a mean proportional between the triangles $PQR$ and $CTT'$.

47. If $CP$ and $CD$ be equal conjugate diameters of an ellipse, and the tangent and normal at $P$ meet the major axis in $T$ and $G$ respectively, prove that $TC \cdot TG = 2CP^2$.

48. An ellipse is inscribed in a triangle having its centre at the centre of the circumscribed circle of the triangle; prove that the perpendiculars from the corners of the triangle on the opposite sides will be normals to the ellipse.

49. Tangents are drawn from any point on the auxiliary circle to the ellipse; prove that the line joining one of the points of contact with one of the foci is parallel to the line joining the other point of contact with the other focus.

50. From a point $P$ without an ellipse, $PQ$ is drawn parallel to the major axis, $Q$ being either of the points in which it meets the curve; then the straight line bisecting $PQ$ at right angles, the tangent at $Q$, and the line which joins the middle points of $PK$, $PL$ (the tangents drawn from $P$) meet in a point.

51. $CP$ and $CD$ are conjugate semi-diameters of an ellipse; $PQ$ is a chord parallel to one of the axes; shew that $DQ$ is parallel to one of the straight lines which join the ends of the axes.

52. A series of ellipses pass through the same point and have a common focus and their major axes of the same length; prove that the locus of their centres is a circle.

What are the limits of the eccentricities of the ellipses?

53. A parallelogram is inscribed in an ellipse, and from any point on the ellipse two straight lines are drawn parallel to the sides of the parallelogram; prove that the rectangles under the segments of these straight lines, made by the sides of the parallelogram, will be to one another in a constant ratio.

54. If the tangent and normal in an ellipse meet the axis in $T$ and $G$ respectively, and $Q$ be the corresponding point of the auxiliary circle, then

$$TQ : TP :: BC : PG.$$

55. $PCP'$ is a diameter of an ellipse, $CD$ conjugate to $CP$; prove that $PD$, $DP'$ are inversely proportional to the diameters which bisect them.

56. If $EF$ be one side of a parallelogram described about an ellipse, having its sides parallel to conjugate diameters, and the lines joining $E$, $F$ to the foci intersect in $O$, $O'$, shew that $O$, $O'$ and the foci will lie on a circle.

57. If the normal at any point of a hyperbola meet the conjugate axis in $g$, and $S$ be the focus, $Pg$ will be to $Sg$ in a constant ratio.

58. In the rectangular hyperbola, if circles be described passing through the centre of the hyperbola and the points of intersection of the tangent with the pairs of conjugate diameters, their centres will lie on a fixed straight line.

59. A chord of a rectangular hyperbola is drawn perpendicular to a fixed diameter and the extremities of the chord and diameter are joined by four straight lines; then the line joining the two fresh points of intersection of these lines will be parallel to a fixed line.

60. A series of confocal ellipses is cut by a confocal hyperbola; prove that either focal distance of any point of intersection is cut by its conjugate diameter with respect to the particular ellipse in a point which lies on a circle.

61. If from any point on the hyperbola a tangent be drawn to the circle described on the transverse axis as diameter, its length is equal to the semi-minor axis of the confocal ellipse through the point.

62. With one focus of a given hyperbola as focus, and any tangent to the hyperbola as directrix, is described another hyperbola touching the conjugate axis of the former; prove that the two will be similar.

63. Two tangents are drawn to the same branch of a rectangular hyperbola from an external point; prove that the angles which these tangents subtend at the centre are respectively equal to the angles which they make with the chord of contact.

64. A circle is described through $P$, $P'$, the extremities of any diameter of a rectangular hyperbola, and cutting the tangent at $P$ and $T$; prove that $P'T$ and the tangent to the circle at $P$ meet on the hyperbola.

65. A fixed hyperbola is touched by a concentric ellipse. If the curvatures at the point of contact are equal, the area of the ellipse will be constant.

66. Two equal circles touch a rectangular hyperbola at a point $O$, and intersect it again in $P$, $Q$; $P'$, $Q'$ respectively; prove that the points may be so taken that $PP'$, $QQ'$ each subtend a right angle at $O$, and that the straight lines joining $P'$, $Q'$ to the centre of the circle $OPQ$ will trisect $OP$, $OQ$ respectively.

67. In a central conic given a focus, the length of the transverse axis, and that the second focus lies on a fixed straight line; prove that the conic will touch two fixed parabolas having the given focus for focus.

68. In a hyperbola a circle is described about a focus as centre, with radius one-fourth of the latus rectum; prove that the focal distances of the points of intersection with the hyperbola are parallel to the asymptotes.

69. Prove that the angles subtended at the vertices of a rectangular hyperbola by any chord parallel to the conjugate axis are supplementary.

70. A line moves in such a manner that the sum of the spaces on its distances from two given points is constant: prove that it always touches an ellipse or hyperbola, the square on whose transverse axis is equal to twice the sum of the squares on the distances of the moving line from the given points.

71. The eccentricity of a hyperbola is 2. A point $D$ is taken on the axis, so that its distance from the focus $S$ is equal to the distance of $S$ from the further vertex $A'$, $P$ being any point on the curve, $A'P$ meets the latus rectum in $K$. Prove that $DK$ and $SP$ intersect on a certain fixed circle.

72. In a hyperbola if $PH$, $PK$ be drawn parallel to the asymptotes, and a line through the centre meet $PH$, $PK$ in $R$, $T$, and the parallelogram $PRQT$ be completed, $Q$ is a point on the hyperbola.

73. If an ellipse and hyperbola have their axes coincident and proportional, points on them equidistant from one axis have the sum of the squares on their distances from the other axis constant.

74. In the hyperbola prove that (*see fig. Prop. XXIII.*)

$$MD : PN :: AC : BC :: CN : CM.$$

75. A parabola and hyperbola have the same focus and directrix, and $SPQ$ is a line through the focus $S$ to meet the parabola in $P$, and the nearer branch of the hyperbola in $Q$; prove that $PQ$ varies as the rectangle contained by $SP$ and $SQ$.

76. If two hyperbolas have their transverse axes parallel, and their eccentricities equal, they will have parallel asymptotes. Does the converse hold?

77. $PP'$ is any diameter of a rectangular hyperbola, $Q$ any point on the curve, $PR$, $P'R'$ are drawn at right angles to $PQ$, $P'Q$ respectively, intersecting the normal at $Q$ in $R$, $R'$: prove that $QR$ and $QR'$ are equal.

78. In a hyperbola a line parallel to $BC$, through the intersection of the tangent at $P$ with the asymptote, meets $DP$, $CA$ produced in the same point, $CD$ and $CP$ being conjugate.

79. From a given point in a hyperbola draw a straight line such that the segment intercepted between the other intersection with the hyperbola and a given asymptote shall be equal to a given line. When does the problem become impossible?

80. A tangent is drawn from any point on the transverse axis of a hyperbola to the auxiliary circle; if the angle between this tangent and the ordinate be called the eccentric angle, shew that the eccentric angles at the points of intersection of an ellipse with two hyperbolas, confocal with itself, are equal.

81. Two unequal parabolas have a common focus and axes opposite; a rectangular hyperbola is described touching both parabolas and having its centre at the common focus; prove that the angle between the lines joining the points of contact to the common focus is 60°.

82. The tangent and normal at any point of a hyperbola intersect the asymptotes and axes respectively in four points which lie on a circle passing through the centre of the hyperbola; and the radius of this circle varies inversely as the perpendicular from the centre upon the tangent.

83. Prove that the rectangular hyperbola which has for its foci the foci of an ellipse, will cut the ellipse at the extremities of the equal conjugate diameters.

84. $ABC$ is a given triangle. $CA$, $CB$ produced are the asymptotes of a hyperbola cutting $AB$ in $P$. Find the position of $P$ when the sum of the squares on its axes is the greatest possible.

85. From the point of intersection of an asymptote and a directrix of a hyperbola a tangent is drawn to the curve; prove that the line joining the point of contact with the focus is parallel to the asymptote.

86. Find the position and magnitude of the axes of a hyperbola which has a given line for asymptote, passes through a given point, and touches a given straight line in a given point.

87. Prove that a hyperbola can be described passing through the extremities of any two diameters of a given ellipse, having diameters conjugate to these for its asymptotes.

88. $PQ$ is a normal at a point $P$ of a rectangular hyperbola meeting the curve again in $Q$; prove that $PQ$ is equal to the diameter of curvature at $P$.

89. $P$ is a point on a hyperbola whose foci are $S$ and $H$; another hyperbola is described whose foci are $S$ and $P$, and whose transverse axis is equal to $SP - 2PH$; shew that the hyperbolas will meet only at one point, and that they will have the same tangent at that point.

90. The tangent at any point on a hyperbola is produced to meet the asymptotes, thus forming a triangle; determine the locus of the point of intersection of the straight lines drawn from the angles of this triangle to bisect the opposite sides.

91. The extremities of the latera recta of all conics which have a common major axis lie on two parabolas.

92. Having given three points, prove that there are four straight lines such, that with any one of them as directrix, and any one of the given points as focus, a conic section may be described passing through the other two given points.

93. In any conic section if $PG$, $pg$, the normals at the ends of a focal chord intersect in $O$, the straight line through $O$ parallel to $Pp$ bisects $Gg$.

94. The normal at $P$ to a central conic meets the axes in $G$ and $g$; $GK$ and $gk$ are perpendicular to the focal distance $SP$; then $PK$ and $Pk$ are constant.

If $kl$ parallel to the transverse axes meet the normal at $P$ in $C$, then $kl$ will be constant.

95. A focal chord $PSQ$ of a conic is produced to meet the directrix in $K$, and $KM$, $KN$ are drawn through the feet of the ordinates $PM$, $QN$ of $P$ and $Q$.

If $KN$ produced, meet $PM$ produced, in $R$, prove that $PR$ is equal to $PM$.

96. The normals at the extremities of a focal chord $PSQ$ of a conic intersect in $K$, and $KL$ is drawn perpendicular to $PQ$; $KF$ is a diagonal of the parallelogram of which $SK$, $SL$ are adjacent sides; prove that $KF$ is parallel to the transverse axis of the conic.

97. With any point on a given circle as focus and a given diameter as directrix, is described a conic similar to a given conic; prove that all such conics will touch the two similar conics to which the given diameter is a latus rectum.

98. Every conic section passing through the centres of the four circles which touch the sides of a triangle, is a rectangular hyperbola; and the locus of the centre of this system of rectangular hyperbolas is the circle circumscribing the triangle.

99. Find the locus of the centres of plane sections of a right cone drawn through a fixed point on the axis of the cone.

100. Shew how to cut a given cone, so that the section may be a parabola of given latus rectum.

101. If sections of a right curve be made, perpendicular to a given plane containing the axis, so that the distance between a focus of a section and that vertex which lies on the same generating line in the given plane be constant, prove that the transverse axes, produced if necessary, of all the sections will touch one of two fixed circles.

102. Find the position of the vertex and axis of a cone of given vertical angle, in order that a given parabola may be a section of the cone.

103. Two right cones have a common vertex; shew how to cut them by a plane so that the sections may be similar and similarly situated curves (1) when they intersect (2) when they do not intersect.

104. The vertex of a cone and the centre of a sphere inscribed within it, are given in position: a plane section of the cone at right angles to any generating line of the cone, touches the sphere; prove that the locus of the point of contact is a surface generated by the revolution of a circle, which touches the axis of the cone at the centre of the sphere.

105. What difference is there in the problems of cutting a given ellipse and a given hyperbola from a right cone?

An ellipse and hyperbola are so situated that the vertices of each curve are the foci of the other, and the curves are in planes at right angles to each other. If $P$ be a point on the ellipse and $Q$ a point on the hyperbola, $S$ the focus, and $A$ the interior focus of that branch of the hyperbola, then

$$PQ + AS = PS + AQ.$$

106. Two cones, whose vertical angles are supplementary, are placed with their vertices coincident and their axes at right angles, and are cut by a plane perpendicular to the common generating line; prove that the directrices of the section of one cone pass through the foci of the section of the other.

# APPENDIX.

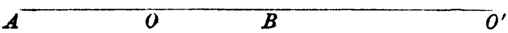
## HARMONIC RATIO.

### POLES AND POLARS.

### RECIPROCATION.

72. Def. If a straight line $AB$ be divided internally in a point $O$, and externally in a point $O'$ so that

$$AO : OB :: AO' : O'B$$



then $AB$ is said to be *harmonically* divided in $O$ and $O'$.

The point $O'$ will evidently be on $AB$ produced through the point $B$ or $A$, according as $AO$ is greater or less than $OB$.

If $AO'$ is equal to $OB$, the point $O'$ moves off to an infinite distance.

It is easy to see that the definition of Harmonic ratio given above coincides with the Algebraical Definition.

For if $AO', OO', BO'$ be three quantities in Harmonic Progression, then by the Algebraical Definition

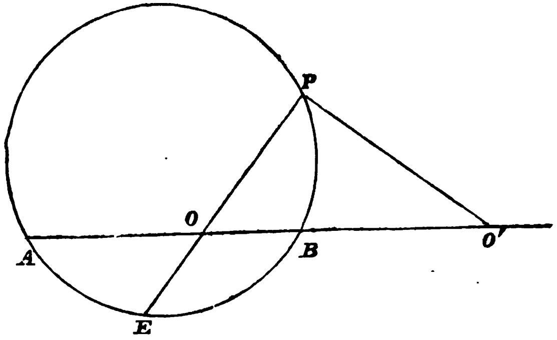
$$\begin{aligned} AO' : O'B &:: AO' \backsim OO' : OO' \backsim BO' \\ &:: AO : BO \end{aligned}$$

N.B.—In Algebraical treatises 3 quantities are said to be in Harmonic Progression when the 1st is to the 3rd as the difference between the 1st and 2nd is to the difference between the 2nd and 3rd, from which definition it at once follows that the reciprocals of the quantities are in Arithmetical Progression.

## PROP. I.

Having given the point $O$, to determine the point $O'$,

Upon $AB$ describe a segment of the circle containing *any* angle.



Bisect either portion of the circumference $AB$ in the point $E$; join $EO$, and produce it to meet the other portion of the circumference in the point $P$.

Draw $PO'$ at right angles to $OP$, meeting $AB$ produced in $O'$; then

$O'$ is the point required.

Since $E$ is the middle point of the arc $AB$,

$\therefore$ the angle $APB$ is bisected by $PO$,

$\therefore$ $PO'$ bisects the *supplementary* angle formed by producing $AP$;

$\therefore$ $AO' : O'B :: AO : OB$. (*Euclid*, VI. A.)

COR. If $O'$ is given, then to determine the point $O$ we have simply to divide $AB$ in the ratio of the two given lines $AO'$, $O'B$.

## PROP. II.

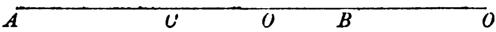
If $AB$ is *harmonically* divided in $O$ and $O'$; then $OO'$ is *harmonically* divided in $B$ and $A$.

Since $AO : OB :: AO' : O'B$,
$\therefore AO : AO' :: OB : O'B$,
or $OB : BO' :: OA : AO'$,
$\therefore OO'$ is harmonically divided in $B$ and $A$.

### PROP. III.

If $AB$ is harmonically divided in $O$ and $O'$, and $AB$ be bisected in $C$; then

$$CO \cdot CO' = CB^2$$



Since $AO : OB :: AO' : O'B$,
$\therefore AO + OB : AO - OB :: AO' + O'B : AO' - O'B$;
or $2CB : 2CO :: 2CO' : 2CB$,
$\therefore CO \cdot CO' = CB^2$.

DEF. When the straight line $AB$ and therefore also $C$ is given points $O$ and $O'$ chosen so that

$$CO \cdot CO' = CB^2$$

are said to be in *involution*. Such points therefore divide $AB$ harmonically.

73. DEF. If a straight line $AB$ be divided in *any* two points $O$ and $O'$, either both internal or both external, or one internal and the other external, and without any limitation as to the side of $AB$ on which the external point or points should fall, the ratio of the ratios $AO : OB$ and $AO' : O'B$, or, as we shall express it,

the ratio $(AO : OB) : (AO' : O'B)$

is called the *an*harmonic ratio of the *range* $AOBO'$, which is always thus represented by $AOBO'$, whatever may be the *geometrical* order in which the letters $O$ and $O'$ appear with reference to $A$ and $B$.

The *an*harmonic ratio of the range $AO'BO$ is

the ratio $(AO' : O'B) : (AO : OB)$,

and is therefore the reciprocal of the *an*harmonic ratio of the range $AOBO'$.

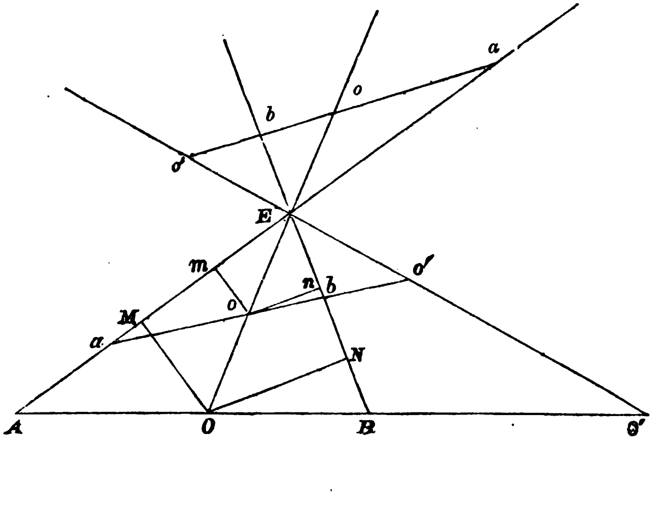
If the points $O$ and $O'$ are both internal or both external with regard to $A$ and $B$, then the *an*harmonic ratio of the range $AOBO'$, viewed *algebraically*, is positive; when one point of division is external and the other internal it is negative. When therefore the straight line $AB$ is *harmonically* divided in $O$ and $O'$, the *an*harmonic ratio of the range $AOBO'$ or $AO'BO$ would be *algebraically* represented by $-1$.

When the *an*harmonic ratios of the ranges $AOBO'$ and $AO'BO$ are equal, it is evident that $AB$ is harmonically divided; for the only value of the *an*harmonic ratio, which can be the same as its reciprocal, except unity (which would imply that the points $O$ and $O'$ were coincident) is $-1$, and in this case, the line $AB$ is harmonically divided.

If the points $A, O, B, O'$ of a range $AOBO'$ be joined to a point $E$, external to $AB$, then the four straight lines thus formed are called a *pencil*, which is represented by $E\,(AOBO')$.

## Prop. IV.

74. If the four straight lines $EA$, $EO$, $EB$, $EO'$ forming a pencil $E\,(AOBO')$ be cut by *any* other straight line in the points $a$, $o$, $b$, $o'$ respectively, then the *an*harmonic ratio of the range $aobo'$ will be the same as that of the range $AOBO'$, however the straight line $aobo'$ be drawn.



From the points $O$ and $o$ draw $OM$, $om$ at right angles to $EA$, and $ON$, $on$ at right angles to $EB$; then the ratio

$$(AO : OB) : (ao : ob :: (\Delta AOE : BOE) : (\Delta aoE : boE)$$
$$:: (OM . AE : ON . EB) : (om . aE : on . Eb)$$
$$:: (AE : EB) : (aE : Eb).$$

So also for the point $O'$

$$(AO' : O'B) : (ao' : o'b) :: (AE : EB) : (aE : Eb)$$
$$\therefore (AO : OB) : (ao : ob) :: (AO' : O'B) : (ao' : o'b),$$
or $(AO : OB) : (AO' : O'B) :: (ao : ob) : (ao' : o'b)$.

## PROP. V.

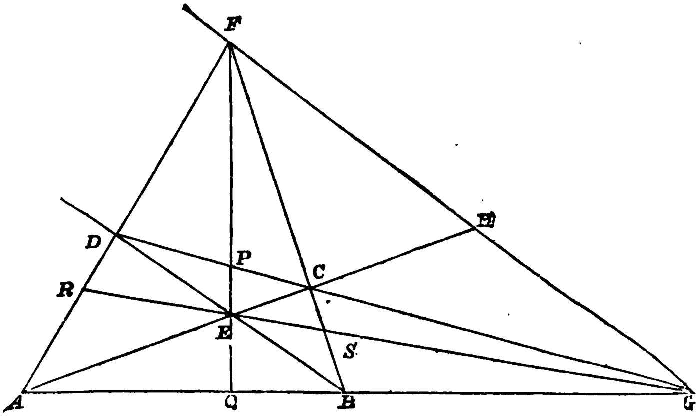
The straight lines joining the intersections of the diagonals of a quadrilateral figure with the points of intersection of a pair of opposite sides are divided harmonically at the points where they meet the other two sides; also the sides of the quadrilateral are divided harmonically by these straight lines.

Let $ABCD$ be a quadrilateral, and let the diagonals $AC$, $BD$, intersect in $E$, and the pairs of opposite sides $AD$, $BC$, and $AB$, $DC$, in $F$ and $G$ respectively.

Let $EF$ meet the sides $CD$, $AB$, in $P$, $Q$; and $EG$ the sides $AD$, $BC$, in $R$, $S$.

Since the four straight lines $EA$, $EQ$, $EB$, $EG$, forming the pencil $E(AQBG)$ are intersected by the straight line $DPCG$ in the points $C$, $P$, $D$, $G$, the ranges $AQBG$, $CPDG$, have the same anharmonic ratio.

Again, since the four straight lines $FA$, $FQ$, $FB$, $FG$, forming the pencil $F(AQBG)$ are intersected by the straight line $DPCG$ in the points $D$, $P$, $C$, $G$, the ranges $AQBG$, $DPCG$, have the same anharmonic ratio.



∴ the ranges $CPDG$, $DPCG$, have the same anharmonic ratio.

But the anharmonic ratio of the range

$$CPDG, \text{ viz. the ratio } (CP : PD) : (CG : GD),$$

is the reciprocal of the anharmonic ratio of the range,

$$DPCG, \text{ viz. the ratio } (DP : PC) : (DG : GC),$$

∴ the ranges $DPCG$ and $CPDG$ are harmonic (*Art.* 73).

Hence also the ranges $RESG$, $AQBG$, are harmonic; and exactly in the same way it may be proved that the ranges $FCSB$, $FPEQ$, $FDRA$, are harmonic.

Cor. If $AC$, $BD$, be produced to meet $FG$ in $H$ and $K$ respectively; then

$AC$, $BD$, are divided harmonically in $E$, $H$, and $E$, $K$ respectively; and

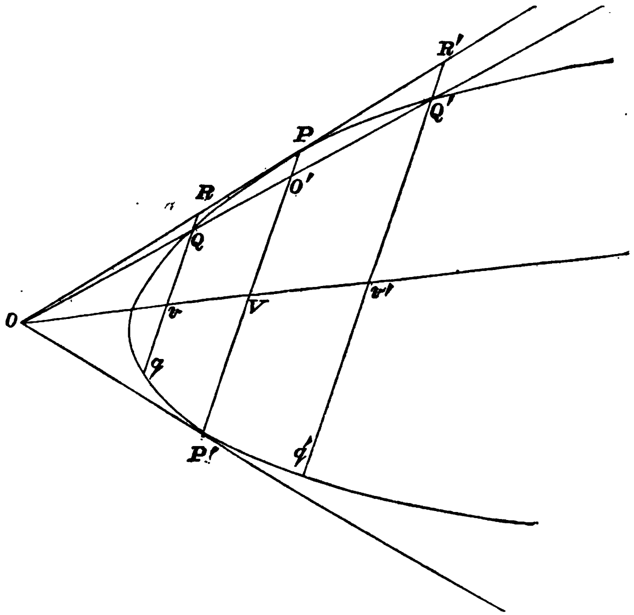
$GF$ is divided harmonically in $H$, $K$.

N.B.—The point $K$ will be on $FG$ or $GF$ produced, according as $GH$ is less or greater than $HF$. If $BE$ be equal to $ED$ the point $K$ is at infinity, and $GF$, $BD$ are parallel, and $GF$ is also bisected in $H$.

## Prop. VI.

75. If from an external point $O$ a pair of tangents $OP$, $OP'$ be drawn to any conic, and a straight line $OQQ'$ intersect the

curve in $Q$, $Q'$ and $PP'$ in $O'$; then $QQ'$ is divided harmonically in $O'$ and $O$.



Through $O$ draw $OV$ bisecting $PP'$ in $V$, and draw the double ordinates $RQvq$, $R'Q'v'q'$ parallel to $PP'$ meeting $OV$ in $v$ and $v'$; then

$Qq$ and $Q'q'$ are bisected in $v$ and $v'$

Now

$$\begin{aligned} RP^2 : R'P^2 &:: RQ \,.\, Rq : R'Q' \,.\, R'q' \\ &:: Rv^2 - Qv^2 : R'v'^2 - Q'v'^2 \\ &:: Ov^2 : Ov'^2 \\ &:: OR^2 : OR'^2 \\ \therefore RP : R'P &:: OR : OR' \\ \therefore QO' : O'Q' &:: QO : OQ' \end{aligned}$$

$\therefore$ $QQ'$ is divided harmonically in $O$, $O'$; and therefore also (see Prop. II.) $OO'$ is divided harmonically in $Q$ and $Q'$.

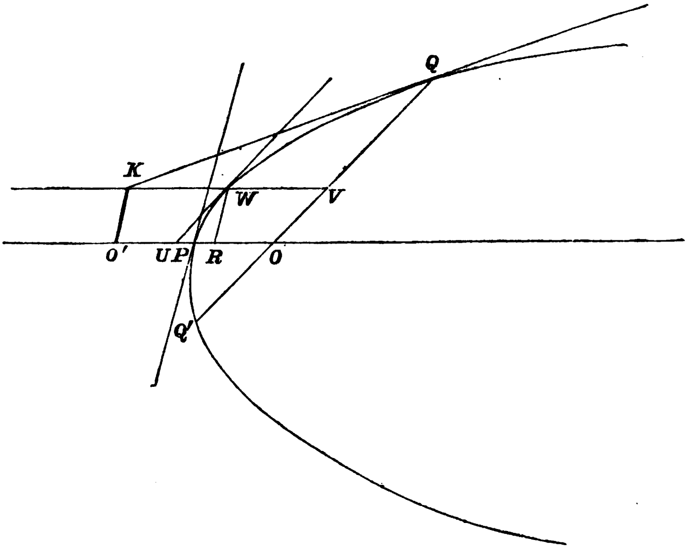
Cor. If the conic is a parabola, $OV$ is drawn parallel to the axis. If an ellipse or hyperbola, $OV$ is drawn through the centre. If the point $O$ be the centre of the hyperbola, then, $OP$, $OP'$ are asymptotes, and the line $PP'$ is *at infinity*, and any chord $QQ'$ through $C$ is bisected at $C$, while the fourth point which with $C$ harmonically divides $QQ'$ is at an infinite distance.

## Prop. VII.

76. The locus of the point of intersection of the tangents at the extremities of *any* chord drawn through a given point either within or without a conic is a straight line.

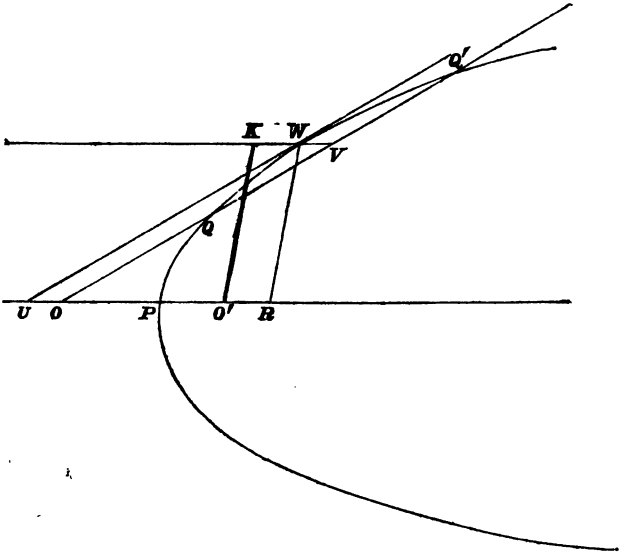
Let $O$ be the given point. Through $O$ draw any chord $QOQ'$, and bisect $QQ'$ in $V$.

First let the curve be a parabola.



Through $O$ and $V$ draw $OPO'$ and $VWK$ parallel to the axis of the parabola meeting the curve in $P$ and $W$.

Make $WK$ equal to $VW$
and $PO'$ equal to $PO$



Then $K$ is the point of intersection of the tangents at $Q$ and $Q'$.

From $W$ draw the tangent $WU$ parallel to $QQ'$ and the ordinate $WR$ parallel to the tangent at $P$ meeting $OO'$ in $U$ and $R$ respectively.

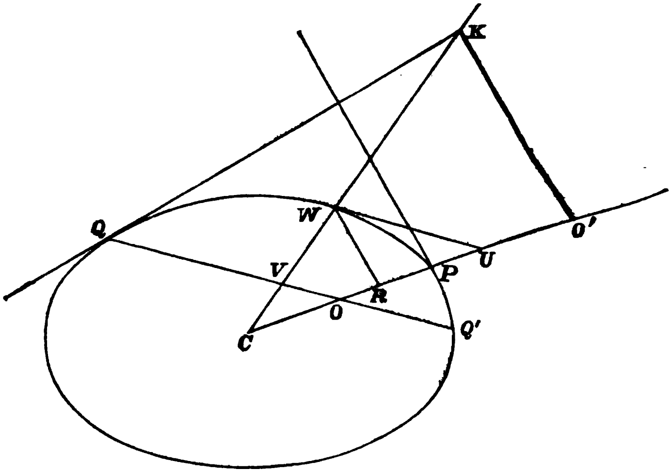
Join $KO'$.

$$\begin{aligned} \text{Now } PO' &= PO \text{ by construction} \\ \text{and } PR &= PU \\ \therefore O'R &= OU \\ &= WV \\ &= KW \end{aligned}$$

$\therefore$ $KO'$ is parallel to $WR$, and therefore to the tangent at $P$.

$\therefore$ the locus of $K$ is the straight line drawn through $O'$ parallel to the tangent at $P$.

Next let the curve be a central conic whose centre is $C$.



Through $C$ draw $COPO'$, $CVWK$ meeting the conic in $P$ and $W$.

Take $CO'$, $CK$ third proportionals respectively to $CO$, $CP$, and $CV$, $CW$, so that

$$CO \,.\, CO' = CP^2 \text{ and } CV \,.\, CK = CW^2.$$

Then $K$ is the point of intersection of the tangents at $Q$ and $Q'$.

Through $W$ draw the tangent $WU$ parallel to $QQ'$, and the ordinate $WR$ parallel to the tangent at $P$ meeting $CP$ in $U$ and $R$ respectively.

Join $KO'$.

$$\begin{aligned} \text{Now } CR \,.\, CU &= CP^2 \\ &= CO \,.\, CO' \\ \therefore\ CR : CO' &:: CO : CU \\ &:: CV : CW \\ &:: CW : CK \text{ by construction.} \end{aligned}$$

$\therefore$ $KO'$ is parallel to $WR$, and therefore to the tangent at $P$.

$\therefore$ the locus of $K$ is the straight line drawn through $O'$ parallel to the tangent at $P$.

N.B.—In the figure the conic has been drawn an ellipse and the point $O$ has been taken on the inside, but it is quite as easy to draw the figure when the point $O$ is on the outside of the curve, or when the conic is a hyperbola, and the point $O$ either internal or external.

77. DEF. The straight line $KO'$ is called the *polar* of the point $O$.

If the point $O$ is on the *concave* side of the conic, the polar is entirely *external*.

When the point $O$ is on the *convex* side of the curve, the polar intersects the conic in two points, and is identical with the chord joining the points of contact of the pair of tangents drawn from $O$.

For if the chord $OQQ'$ drawn through the external point $O$ move round the point $O$ until $Q$, $Q'$ coincide and $OQQ'$ becomes a tangent, the points $K$, $Q$, $Q'$, will evidently all coincide with the point of contact of the tangent drawn from $O$. Hence the points of contact of the tangents drawn from $O$ are on the *polar*, and the *polar* is identical with the chord joining the points of contact.

78. When the pole is given the mode of constructing geometrically the polar, or, when the polar is given, of finding the pole is evident at once from the above proposition, whether the curve be a parabola or a central conic.

### PROP. VIII.

If the polar of $O$ pass through $o$, then the polar of $o$ passes through $O$.

The points $O$ and $o$ may be either both *external*, or one *internal* and the other *external*, but they cannot be both internal; for the polar of an *internal* point is wholly *external*.

(1) Let $O$ be external. Then the polar of $O$ is the chord joining the points of contact of tangents drawn from $O$; and since this chord passes through $o$, and $O$ is the point of intersection of the tangents at the extremities of this chord, $O$ is evidently on the polar of $o$.

(2) Let $O$ be internal. Then since the point $o$ is on the polar of $O$, the chord of contact of the pair of tangents drawn from $o$, *i.e.* the polar of $o$, will pass through the point $O$.

COR. From this it is evident that the point of intersection of two polars to a conic is the pole of the line joining the two poles.

### PROP. IX.

Any chord of a conic is divided harmonically by any point upon it, and the point where the polar of this point meets the chord.

Let $QQ'$ be any chord of a conic, and $O$ any point upon it.

(1) If the point $O$ is *external* this proposition is already proved by Prop. VI.

(2) If the point $O$ is on the concave side of the conic, let the polar of $O$ intersect $QQ'$ or $QQ'$ produced in $O'$; then since the point $O'$ is on the convex side of the curve, and is on the polar of $O$, the polar of $O'$ *i.e.* the chord joining the points of contact of the pair of tangents drawn from $O'$ must pass through $O$, and therefore as before $QQ'$ is divided harmonically in $O$ and $O'$.

79. If any number of points be on a straight line, their polars with respect to any conic will pass through the same point, viz., the pole of the straight line on which the points all lie; and conversely if any number of straight lines pass through a point, their poles all lie on the same straight line, viz., the polar of the given point. This is briefly expressed by saying that if any number of points are *collinear*, their polars are *confocal*, and conversely if any number of lines are *confocal*, their poles are *collinear*.

If a point instead of moving upon a straight line trace out a curve, the polars of the various points of the curve with respect to any conic (which in this case is called the *auxiliary* conic) will by their ultimate intersections form a new curve to which they are all tangents.

Thus if $P$ and $Q$ be two contiguous points on the original curve and $p$ and $q$ represent their polars (with respect to any auxiliary conic) the intersection of the straight lines $p$ and $q$ will *ultimately* be a point on the new curve; and since this point will be the pole of $PQ$ which when $P$ and $Q$ are close together is *ultimately* a tangent to the original curve, it is evident that the polars of the several points of the new curve are tangents of the original curve, which can therefore be derived from the second curve in exactly the same manner as the second was derived from the first.

On account of this reciprocal property the locus of the ultimate intersections or the envelope of (*i.e.* the curve touched by) the polars of the various points of a given curve, is called the polar reciprocal of the proposed curve.

In *analytical* treatises it is shown that the polar reciprocal of any conic (with respect to an auxiliary conic) is also a conic.

In this brief notice we shall confine ourselves to shewing that any conic section may be produced by reciprocating, *i.e.* taking the polar reciprocal of a *circle* with respect to an *auxiliary* circle.

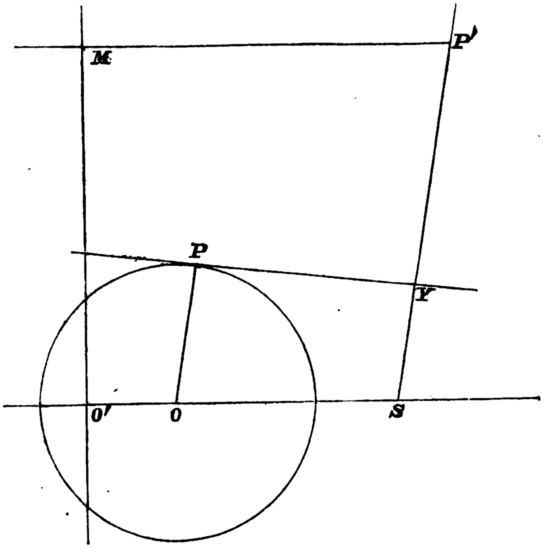
The advantage of having a circle for the *auxiliary* conic consists in the fact that the straight line drawn from any point to the centre of the circle is perpendicular to the polar of the point, and therefore also that the angle subtended at the centre of the circle by any two points is equal to one of the angles contained by the polars of the points.

In finding the polar reciprocal of a given curve, there are two ways in which we may proceed. We may either find the envelope of the polars of the different points of the original curve, or we may find the locus of the poles of the tangents at the different points of the original curve. The latter is the method which will be followed in the next article.

### PROP. X.

The polar reciprocal of a circle, with respect to an auxiliary circle, will be a conic.

Let $S$ be the centre of the auxiliary circle, and $O$ the centre of the circle to be reciprocated.



Join $SO$, and on $SO$ produced, if necessary, take $SO'$ a third proportional to $SO$ and the radius of the auxiliary circle. Through $O'$ draw $O'M$ at right angles to $SO'$, then $O'M$ is the polar of $O$ (Art. 78).

At any point $P$ on the circle whose centre is $O$, draw the tangent $PY$; and from $S$ draw $SY$ perpendicular to $PY$.

On $SY$ produced, if necessary, take a point $P'$ so that

$$SY \,.\, SP' = SO \,.\, SO'$$

Then $P'$ is the pole of $PY$.

Draw $P'M$ perpendicular to $O'M$; then since

$$SY \,.\, SP' = SO \,.\, SO'$$

$$\therefore SP' : SO' :: SO : SY$$

Now since the quadrilateral figures $SP'MO'$, $SOPY$ are equiangular and

$$SP' : SO' :: SO : SY$$

it is at once seen that these figures are also *similar*.

$$\therefore SP' : P'M :: SO : OP$$

$\therefore$ the locus of $P'$ is a conic whose focus is $S$, directrix $O'M$ the polar of $O$, and eccentricity the ratio of $SO : OP$.

Cor. Since the eccentricity of the conic depends only upon the ratio of $SO : OP$, the polar reciprocal will be an ellipse, parabola, or hyperbola, according as $S$ is inside, on, or outside the circle to be reciprocated.

The distance of the directrix from $S$ can be made as large or small as we please by increasing or diminishing the radius of the auxiliary circle, without altering the eccentricity of the curve.

If the point $P$ be such that the tangent pass through $S$, the point $P'$ will be at infinity. If therefore tangents be drawn from $S$ to the given circle, the lines drawn through $S$ at right angles to these tangents will meet the curve at infinity, and will therefore be parallel to the asymptotes of the conic, which in this case will be an hyperbola since $S$ is outside the given circle.

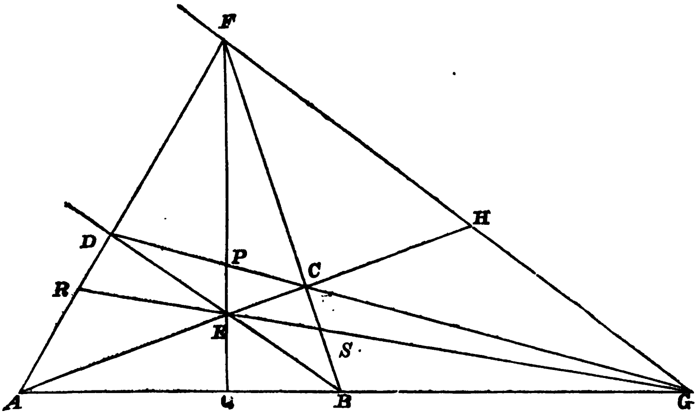
The focus $S$, the distance $SO'$ of the directrix, and the eccentricity being known, all the other elements of the conic can be at once determined.

80. By combining the harmonic properties of the complete quadrilateral with the harmonic property of the conic, viz., that any chord is divided harmonically by the pole and polar, many important theorems with regard to figures inscribed in and circumscribed about a conic may be deduced.

## Prop. XI.

The triangle formed by the point of intersection of the diagonals of a quadrilateral figure inscribed ~~is~~ a conic, and in the points of intersection of the opposite sides is self-conjugate, *i.e.* each angular point is the pole (with regard to the conic) of the opposite side.

Let $ABCD$ be a quadrilateral inscribed in a conic.



Now since the chord $BC$ is divided harmonically in $F$ and $S$, (*Prop.* V.)

$\therefore$ the polar of $F$ passes through $S$.

So the polar of $F$ passes through $R$,

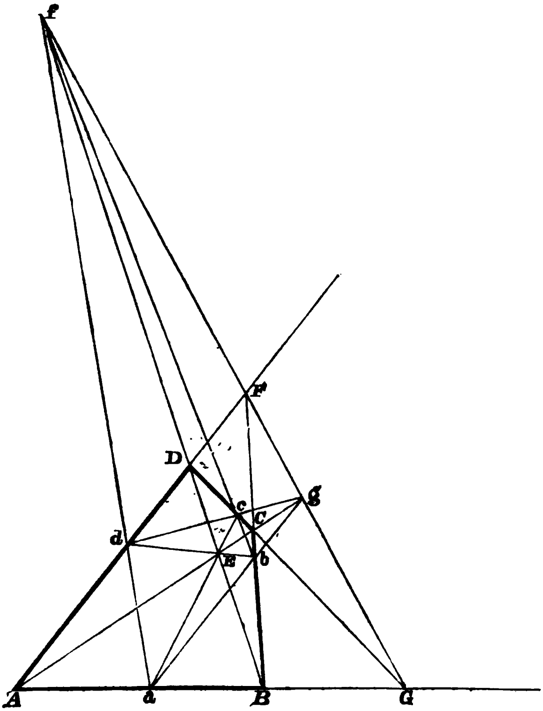
$\therefore$ $RS$ or $EG$ is the polar of $F$.

So also $EF$ is the polar of $G$.

Hence $E$ the point of intersection of $EF$ and $EG$ is the pole of $FG$ and $\therefore$ the triangle $EFG$ is self-conjugate.

## PROP. XII.

The intersection of the diagonals of a quadrilateral figure described about a conic is the pole with respect to the conic of the straight line joining the points of intersection of the pairs of opposite sides.



Let $ABCD$ be a quadrilateral figure described about a conic; then the intersection $E$ of the diagonals $AC$, $BD$ shall be the pole of the straight line $FG$ joining the points of intersection of $AD$, $BC$ and $AB$, $DC$.

Let $a$, $b$, $c$, $d$ be the points of contact of the sides of the quadrilateral with the conic.

Complete the quadrilateral $abcd$, and let $e$ be the intersection of the diagonals $ac$, $bd$; and $f$ and $g$ those of the pairs of the opposite sides $ad$, $bc$ and $dc$, $ab$ respectively.

Now by the last proposition $e$ is the pole of $fg$ with respect to the conic which is inscribed in $ABCD$ and therefore described about $abcd$.

But since $F$ is the pole of $db$ and $G$ is the pole of $ac$

$\therefore$ $e$ is the pole of $FG$

$\therefore$ the straight lines $fg$ and $FG$ coincide.

Again since $A$ is the pole of $ad$ and $C$ is the pole of $bc$

$\therefore$ $f$ is the pole of $AC$

So also $g$ is the pole of $BD$

$\therefore$ $E$ is the pole of $fg$

$\therefore$ the points $E$ and $e$ coincide,

but $e$ is the pole of $FG$

$\therefore$ $E$ is the pole of $FG$.

COR. 1. Since $f$ is the pole of $AC$ and also of $eg$ or $Eg$,
the straight line $AC$ passes through $g$
So also $BD$ passes through $f$.

COR. 2. If $ac$ pass through the point $F$ then $G$ is the pole of $EF$, and

$E$ has been proved to be the pole of $FG$

$\therefore$ $F$ is the pole of $EG$

$\therefore$ $bd$ passes through the point $G$.

In this case, but in this only, the $\Delta\, EFG$ is self-conjugate.

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